A game-theoretic market-based approach to extract flexibility from distributed energy resources

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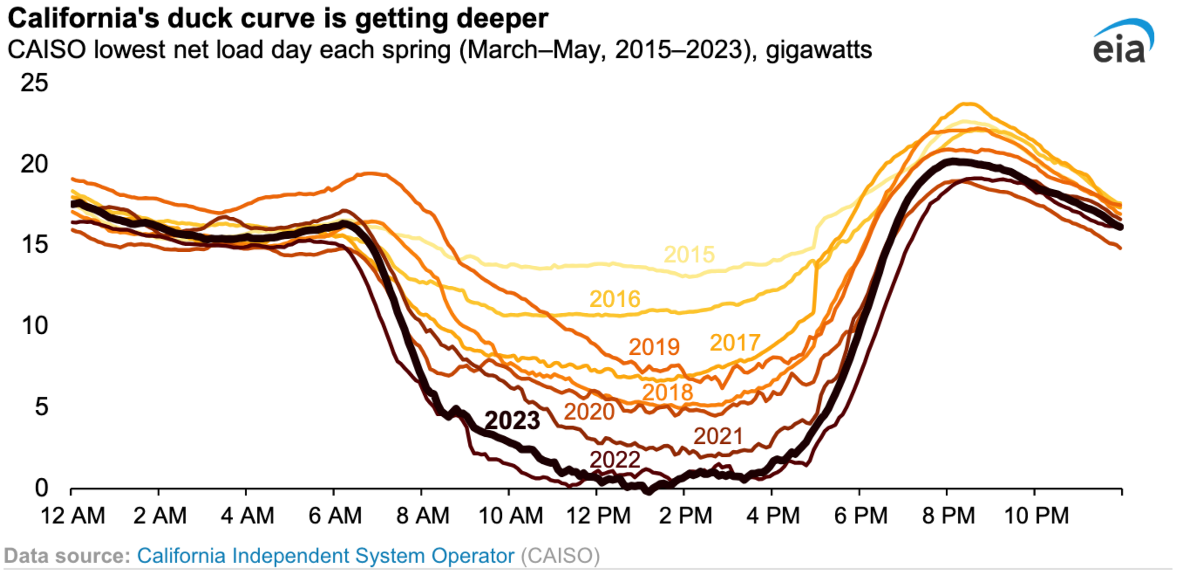
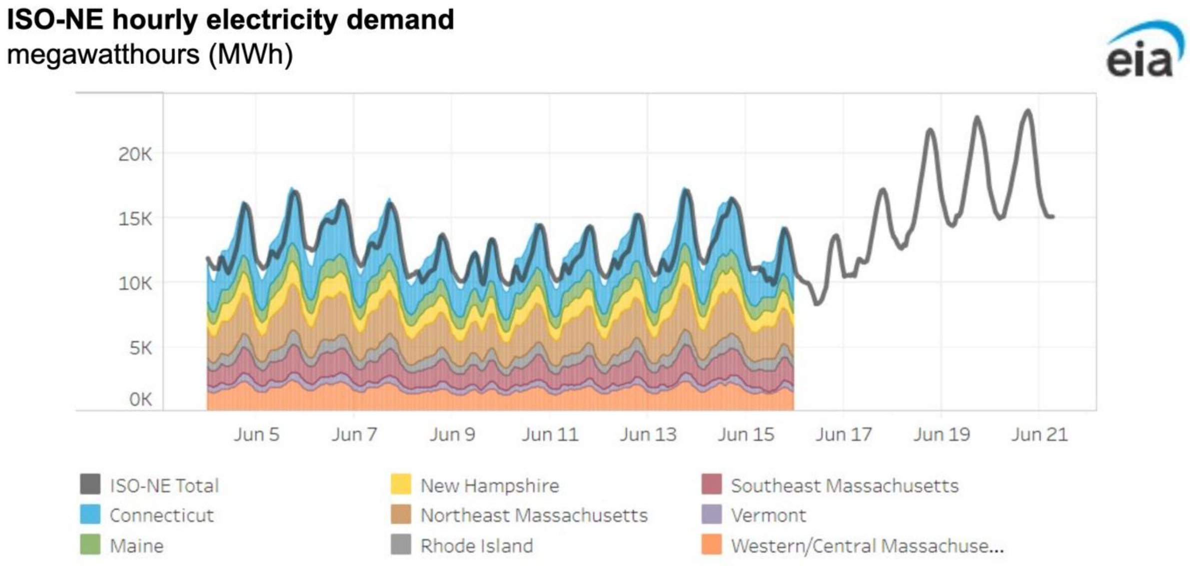
Challenges for the future decarbonized grid

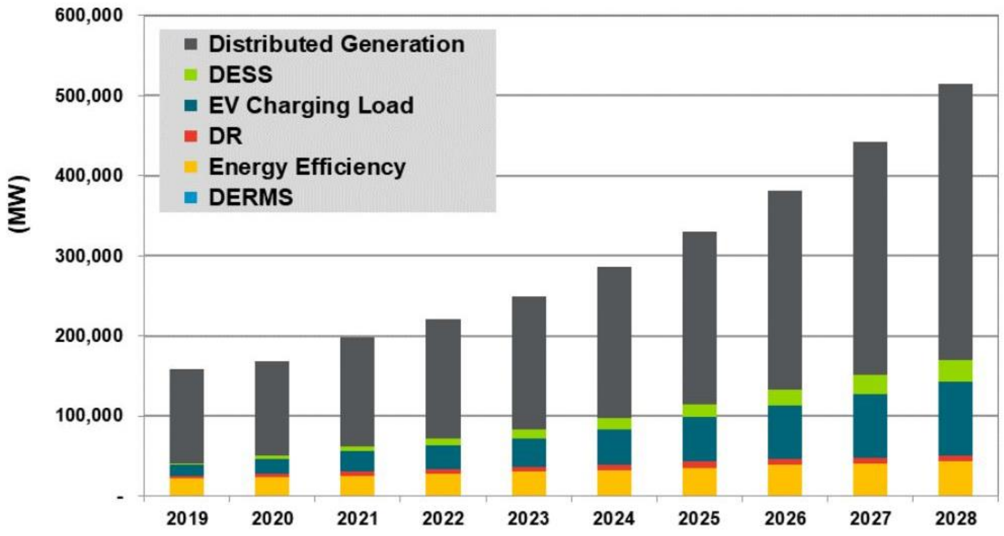
- Intermittency & variability of renewables → Reliability & stability issues
- Rapid load growth (e.g. heat pumps, EVs, data centers) → Stress on grid
- Extreme weather events
- Voltage & frequency issues due to lower inertia

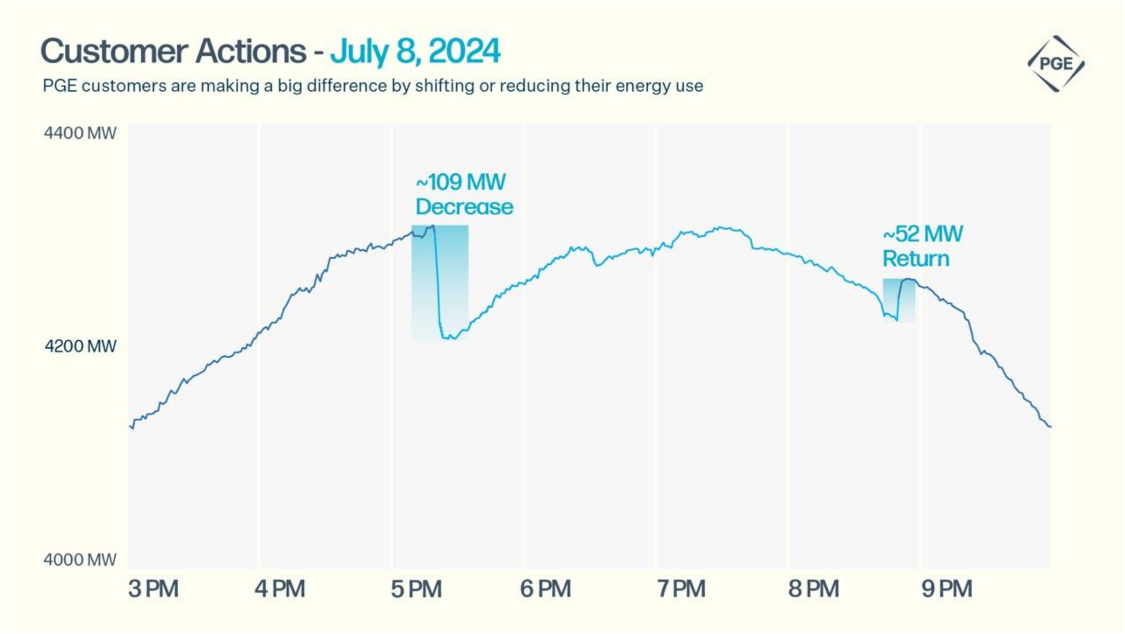




Potential solution: Flexibility

- Distributed energy resources (solar PV, batteries, flexible loads) are flexible
- DERs can provide flexibility as a grid service
- Flexibility can potentially:
  - Improve voltage & frequency profiles
  - Mitigate impacts of extreme weather
  - Improve dispatch efficiency to lower operating costs + prices





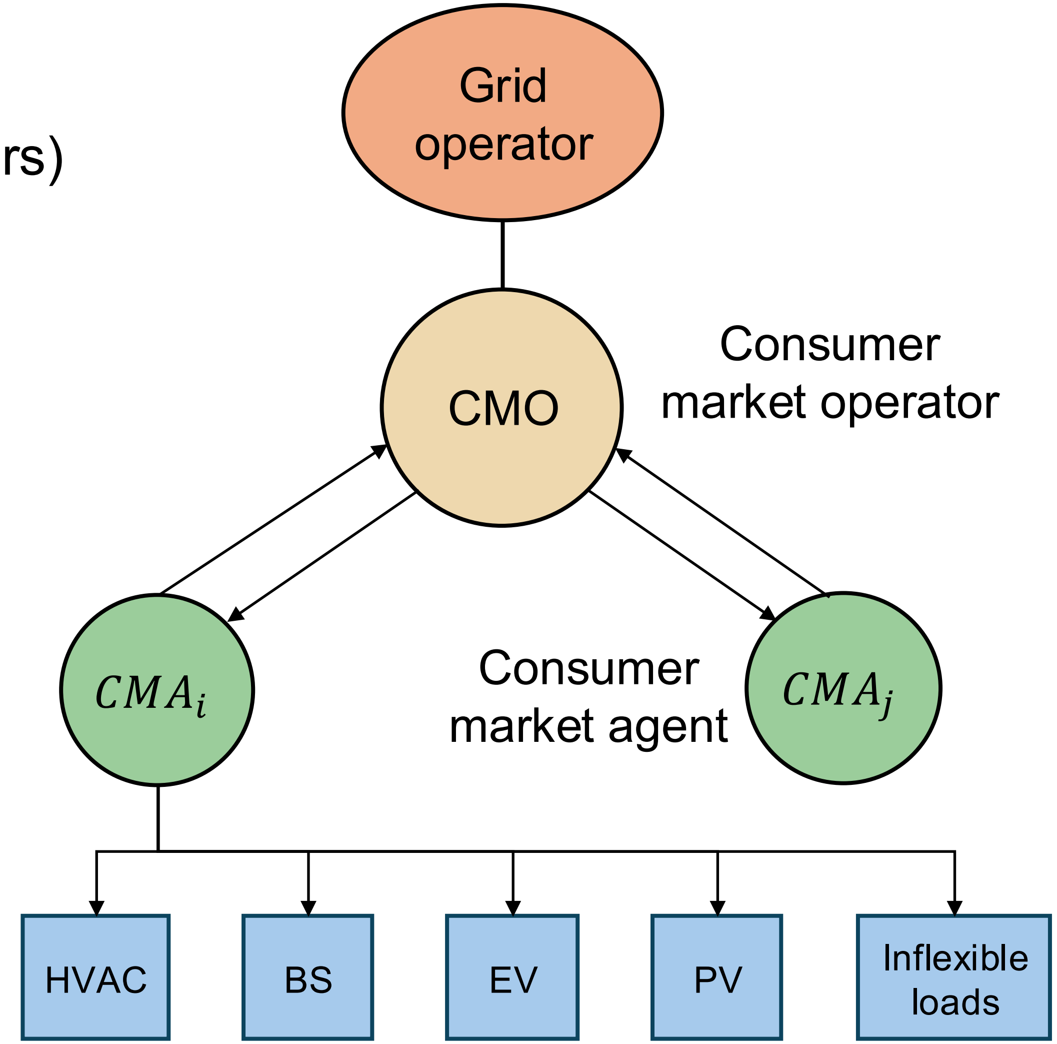
Literature review + contributions

- Rich literature on applications of **game theory** (Saad et al. 2012, Fadlullah et al. 2011), & **mechanism design** (Eid et al. 2016) to electricity markets
- Common modeling tools**: Stackelberg** (Maharjan et al. 2013) & **coordinated (coalitional)** games (Turdybek et al. 2024, Saeian et al. 2022)
- Some have also proposed **distributed algorithms** to solve such games (Li et al. 2011, Anoh et al. 2020) OR utilized **Vickrey-Clarke-Groves** mechanisms (Nekouei et al. 2015)
- We build upon this work:
  1. Analyze how game theory and mechanism design can inform development of markets closest to end-users (electricity consumers & prosumers)
  2. More accurately model physical dynamics & constraints of DERs
  3. Propose new approach to aggregate & maximize flexibility
  4. Using different types of tariffs to charge or compensate agents

Consumer market (CM) formulation

- Formulate as bilevel, multi-stage Stackelberg game between CMO (leader) & CMAs (followers)

1. CMA coordinates its DERs via multiperiod optimization (MPO) to determine its maximum possible flexibility ranges given physical constraints of its DERs
2. CMO solves welfare maximization problem to clear CM & set prices, given all CMA bids
3. Given CM prices, CMAs solve utility maximization problem to determine optimal flexible injection setpoints



Stage I: Multiperiod optimization to coordinate DERs

- Each CMA maximizes net power injection into grid & flexibility extracted from its DERs while minimizing costs & disutility

$$\min_{P_i^d(t),\delta_i^d(t)} \sum_{t\in\mathcal{H}}\sum_{d\in\mathcal{D}} -\delta_i^d(t) + f_i^d(P_i^d) + f_i^{util}(P_i^d) - P_i^{total}(t)$$

$$\text{s.t. } \underline{P_i^d(t)} \le P_i^d(t) - \delta_i^d(t),\ P_i^d(t) + \delta_i^d(t) \le \overline{P_i^d(t)}$$

$$\text{All device-specific state constraints for each DER } d \in \mathcal{D}_i$$

$$P_i^{total}(t) = \sum_{i\in\mathcal{D}_i} P_i^d(t) - P_i^{fixed}(t),\ \epsilon_1|P_i^d| \le \delta_i^d \le \epsilon_2|P_i^d|$$

$$\mathcal{D}_i \subseteq \{BS, EV, HVAC, PV\},\ \epsilon_1 < \epsilon_2$$

Net injection = Generation - Load

- Reformulation of absolute injections for $d \in \{BS, EV\}$

$$P_i^d = P_i^{d,+} - P_i^{d,-},\ |P_i^d| = P_i^{d,+} + P_i^{d,-},\ z_i^d \in \{0,1\}$$

$$0 \le P_i^{d,+} \le z_i^d\overline{P_i^d},\ 0 \le P_i^{d,-} \le (1-z_i^d)|\underline{P_i^d}|$$

$$\text{Note: } |P_i^{HP}| = -P_i^{HP} \text{ and } |P_i^{PV}| = P_i^{PV}$$

Device-specific DER constraints & costs: BS & EV

- State of charge dynamics with inter-temporal constraints & cycling costs

$$SOC_i^{BS}(t+1) = (1-\delta_{BS}^i)SOC_i^{BS}(t) - \frac{P_i^{BS}(t)\Delta t \eta_i^{BS}}{\overline{E}_i^{BS}}$$

$$\underline{P}_i^{BS} \leq P_i^{BS}(t) \leq \overline{P}_i^{BS}, \ \underline{SOC}_i^{BS} \leq SOC_i^{BS}(t) \leq \overline{SOC}_i^{BS}$$

$$SOC_i^{BS}(0) = SOC_i^{BS}(T)$$

$$f_i^{BS}(P_i^{BS}) = \alpha_{cyc} \sum_{t=t_H}^{t_H+(H-1)\Delta t} (P_i^{BS}(t+1) - P_i^{BS}(t))^2$$

- EV model similar to BS
- Tracking objective to achieve desired SOC by specific time (e.g. 90% by 9am)
- Additional restriction when EV is unavailable during time window (e.g. 9am-5pm while at work) $P_i^{EV}(t) = 0 \; \forall \, t \in [t_1, t_2]$

$$f_i^{EV} = \alpha_{cyc} \sum_{t=t_H}^{t_H+(H-1)\Delta t} (P_i^{EV}(t+1) - P_i^{EV}(t))^2 + \xi_{ev}(SOC_i^{EV}(t^*) - SOC_i^{EV^*})^2$$

DER constraints & costs: Heat pump & solar

- **HP**: Thermal dynamics, temperature comfort limits, setpoint tracking

$$\text{Cooling mode } (T_i^{out} > T_i^{in}) : T_i^{in}(t+1) = \theta_i T_i^{in}(t) + (1-\theta_i)\left(T_i^{out}(t) + \rho_i P_i^{HP}(t)\right)$$

$$\theta_i = e^{\frac{-\Delta t}{R_i^{th} C_i^{th}}} \approx 1 - \frac{\Delta t}{R_i^{th}, C_i^{th}}, \rho_i = R_i^{th}\eta_i$$

$$\text{Heating mode } (T_i^{out} < T_i^{in}) : T_i^{in}(t+1) = \theta_i T_i^{in}(t) + (1-\theta_i)\left(T_i^{out}(t) - \rho_i P_i^{HP}(t)\right)$$

$$-P_{rated,i}^{HP} = \underline{P}_i^{HP} \leq P_i^{HP}(t) \leq 0, \ \underline{T}_i^{in} \leq T_i^{in}(t) \leq \overline{T}_i^{in}$$

$$f_i^{HP} = \xi_{ac} \sum_{t=t_H}^{t_H+(H-1)\Delta t} (T_i^{in}(t) - T_i^{in^*})^2$$

- **PV**: Non-dispatchable, can be curtailed
- CMA also utilizes as much PV output as possible (when available) to charge BS & EV, by minimizing this objective:

$$0 \leq P_i^{PV}(t) \leq \alpha^{PV}(t)\overline{P}_i^{PV}$$

$$f_i^{PV} = \xi_{pv} \sum_{t=t_H}^{t_H+(H-1)\Delta t} (\alpha^{PV}(t)\overline{P}_i^{PV} - P_i^{PV}(t))^2$$

$$f_i^{util}(t) = \left(P_i^{PV} + P_i^{BS} + P_i^{EV}\right)^2 \forall \{t : P_i^{PV}(t) \neq 0\}$$

Stage II: CMA welfare maximization problem

- CMA aggregates schedules across all DERs

$$P_i^0 = \sum_{d \in \mathcal{D}_i} P_i^{d^*}, \ \underline{P}_i = P_i^0 - \sum_{d \in \mathcal{D}_i} \delta_i^{d^*}, \ \overline{P}_i = P_i^0 + \sum_{d \in \mathcal{D}_i} \delta_i^{d^*}$$

- Maximize social welfare s.t. flexibility constraints, given prices for electricity $\mu(t)$ & flexibility $\tilde{\mu}(t)$ set by CMO

$$\max_{P_i} \ U_i^{cma}(P_i, \widetilde{\mu}, \mu) = \widetilde{\mu}(P_i - P_i^0) + \mu P_i - \gamma_i (P_i - P_i^0)^2 \text{ s.t. } \underline{P}_i \le P_i \le \overline{P}_i, P_i \ge P_i^0$$

- Analytically solve game via KKT conditions

$$P_i^*(\widetilde{\mu}, \mu) = \begin{cases} \overline{P}_i & \text{if } \frac{\gamma_i(\overline{P}_i - P_i^0)}{\widetilde{\mu} + \mu} < \frac{1}{2} \\ P_i^0 + \frac{\widetilde{\mu}+\mu}{2\gamma_i} & \text{otherwise} \end{cases}$$

- CMA submits optimal bid $\{P_i^0, P_i^*, [\underline{P_i}, \overline{P_i}]\}$ to CMO

Stage III: CMO optimization to set optimal prices

- CMO aims to schedule its CMAs to track desired flexible setpoint $\tilde{P}(t)$ requested by LEM s.t. to budget balance

$$\min_{\widetilde{\mu}(t),\mu(t)} -U^{cmo}(P_i(t),\mu(t),\widetilde{\mu}(t)) = \left(\sum_{i\in\mathcal{C}} P_i(t) - \widetilde{P}(t)\right)^2$$

$$\text{s.t. } P_i^0(t) \le P_i(t) \le \overline{P}_i(t)\ \forall\ i$$

$$\widetilde{\mu}(t)\sum_{i\in\mathcal{C}}(P_i(t) - P_i^0(t)) + \mu(t)\sum_i P_i(t) = \pi(t)\sum_i P_i(t)$$

- Analytically solve for optimal prices for power & flexibility

$$\widetilde{\mu}(P_t - P_t^0) + \mu P_t = \pi P_t,\ P_t^* = \sum_{i\in\mathcal{C}} P_i^* = P_t^0 + \gamma_t\frac{\mu+\widetilde{\mu}}{2} = \widetilde{P} \implies \mu^* = \frac{\pi\widetilde{P}}{P_t^0} - \frac{2(\widetilde{P}-P_t^0)^2}{\gamma_t P_t^0}$$

$$\widetilde{\mu}^* = \frac{P_t^*(\pi-\mu^*)}{P_t^*-P_t^0} = \frac{\widetilde{P}(\pi-\mu^*)}{\widetilde{P}-P_t^0} = \frac{\widetilde{P}\left(2(\widetilde{P}-P_t^0)-\pi\gamma_t\right)}{\gamma_t P_t^0},\quad \gamma_t = \sum_{i\in\mathcal{C}}\frac{1}{\gamma_i}$$

Positivity of prices

- Following condition required for $\mu^*(t), \tilde{\mu}^*(t) > 0$

$$\begin{cases} \widetilde{P} > P_t^0 + \frac{\pi\gamma_t}{2} & \text{if } \widetilde{P}, P_t^0 < 0 \\ \max\left(a_1, P_t^0 + \frac{\pi\gamma_t}{2}\right) \le \widetilde{P} \le a_2 & \text{if } \widetilde{P}, P_t^0 > 0 \end{cases}$$

$$a_1 = P_t^0 + \frac{\pi\gamma_t}{4} - \frac{1}{2}\sqrt{\frac{\pi\gamma_t}{2}\left(4P_t^0 + \frac{\pi\gamma_t}{2}\right)}$$

$$a_2 = P_t^0 + \frac{\pi\gamma_t}{4} + \frac{1}{2}\sqrt{\frac{\pi\gamma_t}{2}\left(4P_t^0 + \frac{\pi\gamma_t}{2}\right)}$$

- Generally holds true given that:
  - CMO is generally a net load → $\tilde{P}, P_t^0 < 0$
  - CMO provides upward flexibility to reduce net load → $\tilde{P} > P_t^0$
  - $\frac{\pi\gamma_t}{2}$ is small for most realistic prices π & disutility coefficients γ_i

Equilibrium

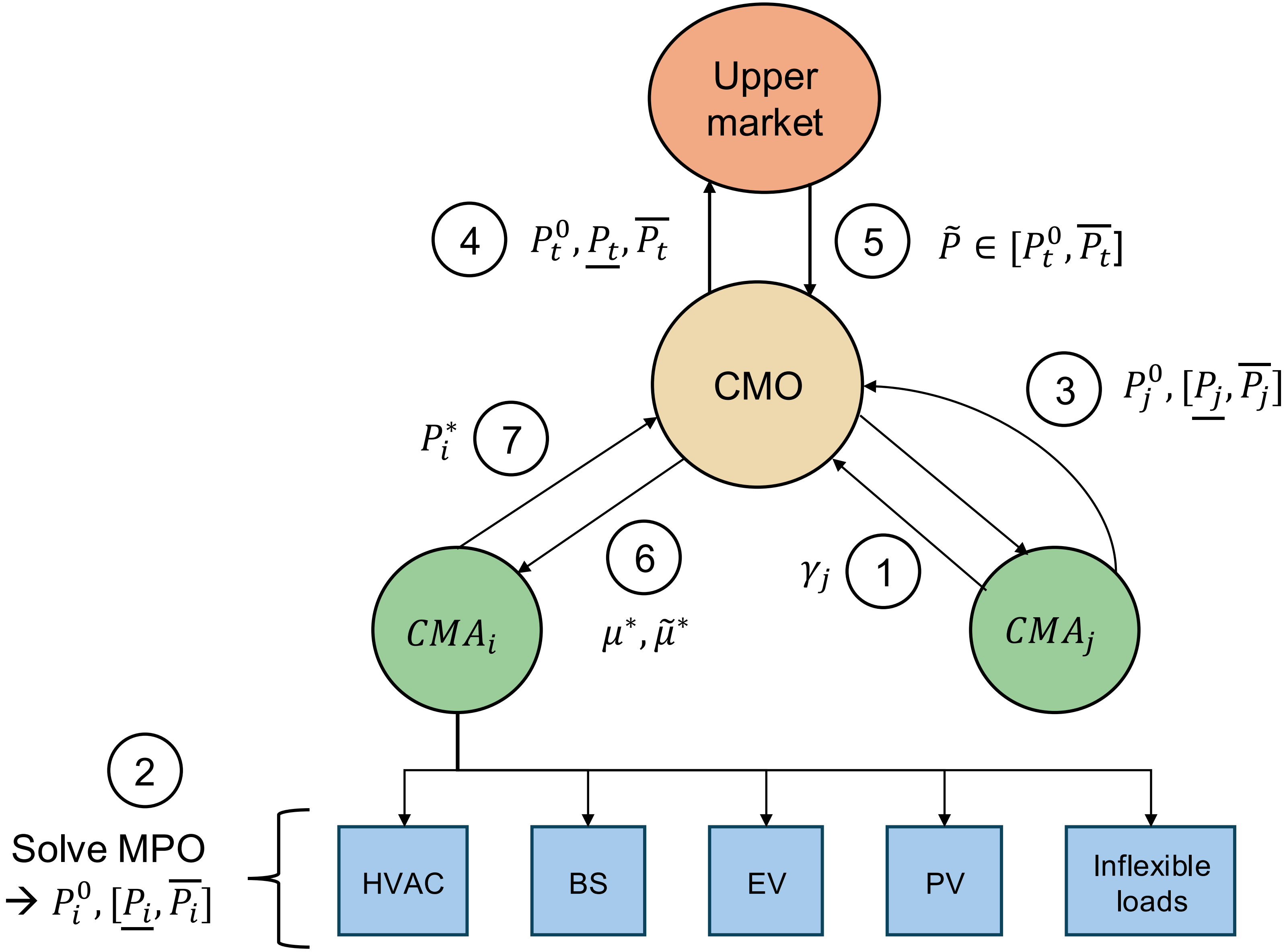
- Note: Nash's theorem states existence of mixed strategy NE for any finite game and Glicksberg's theorem extends this to infinite setting with continuous strategies
- Our assumptions & simplifications allow us to show existence & uniqueness of pure strategy equilibrium with closed form solutions
- Optimal prices $\mu^*, \tilde{\mu}^*$ set by CMO will induce optimal bids P_i^* from all CMAs that lead to an equilibrium in pure strategies
- Set of bids & prices $\{P_i^* \ \forall \ i, \mu^*, \tilde{\mu}^*\}$ correspond to unique **Nash equilibrium** amongst CMAs and a unique **Stackelberg equilibrium** between all CMAs & the CMO

$$U_i^{cma}(P_i, \widetilde{\mu}(P_i, P_{-i}), \mu(P_i, P_{-i})) \equiv U_i^{cma}(P_i, P_{-i}, \widetilde{\mu}, \mu)$$

$$U_i^{cma}(P_i^*, P_{-i}^*, \widetilde{\mu}^*, \mu^*) \geq U_i^{cma}(P_i, P_{-i}^*, \widetilde{\mu}^*, \mu^*) \forall P_i \in [\underline{P}_i, \overline{P}_i]$$

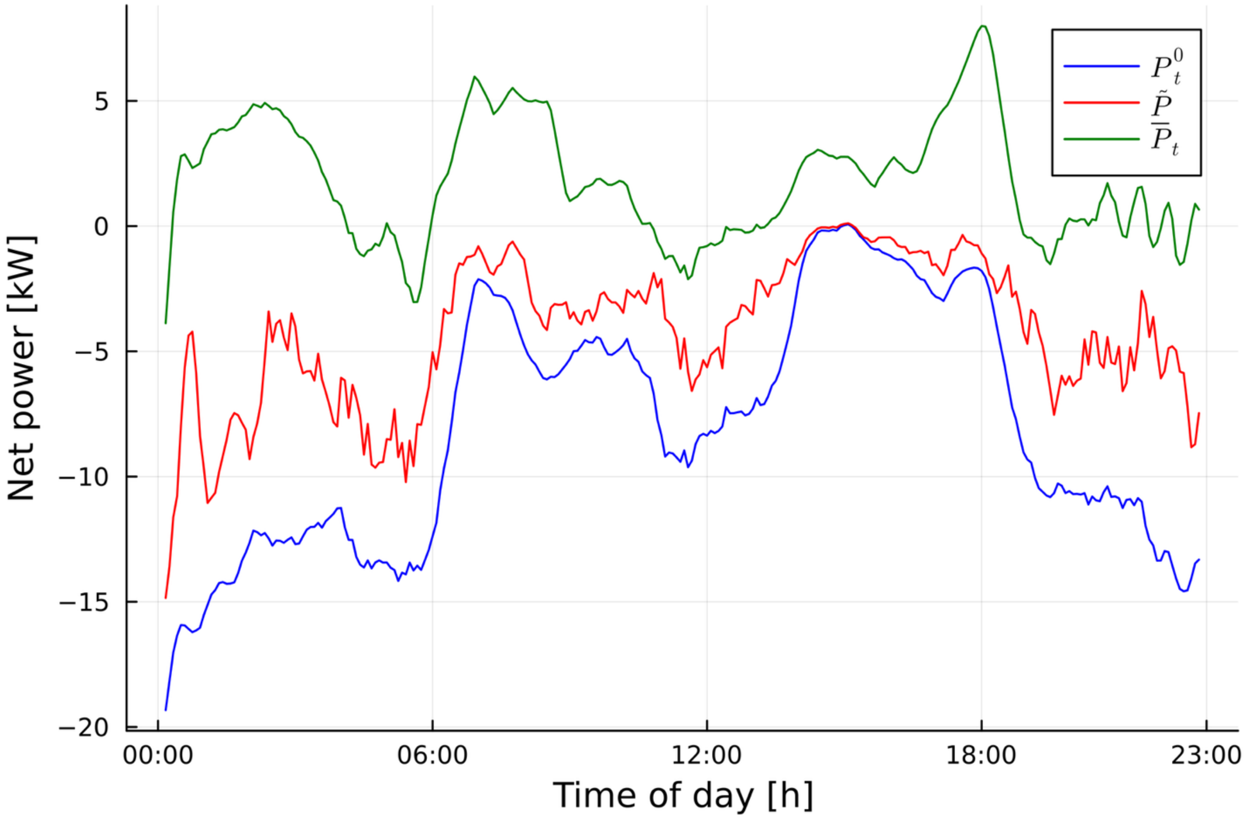
$$U^{cmo}(P_i^*, P_{-i}^*, \mu^*, \widetilde{\mu}^*) = 0 \geq U^{cmo}(P_i^*, P_{-i}^*, \mu, \widetilde{\mu}) \ \forall \mu, \widetilde{\mu}$$

Overall process for CM operation

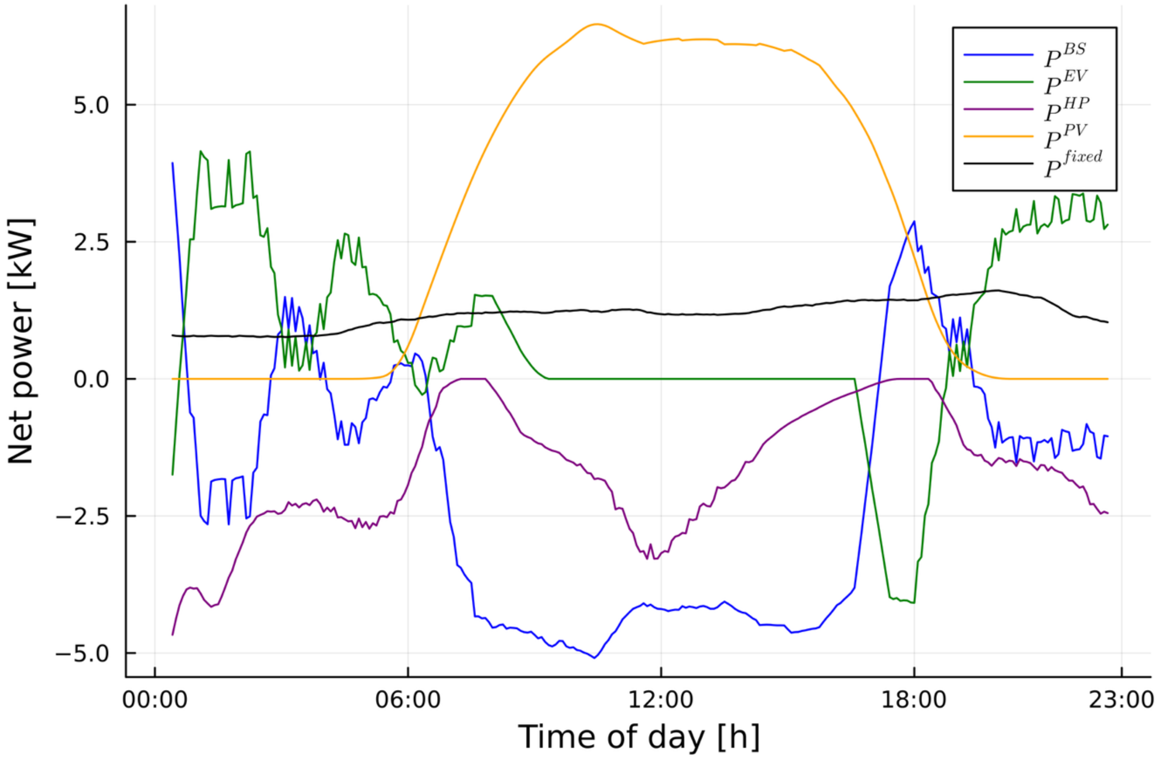


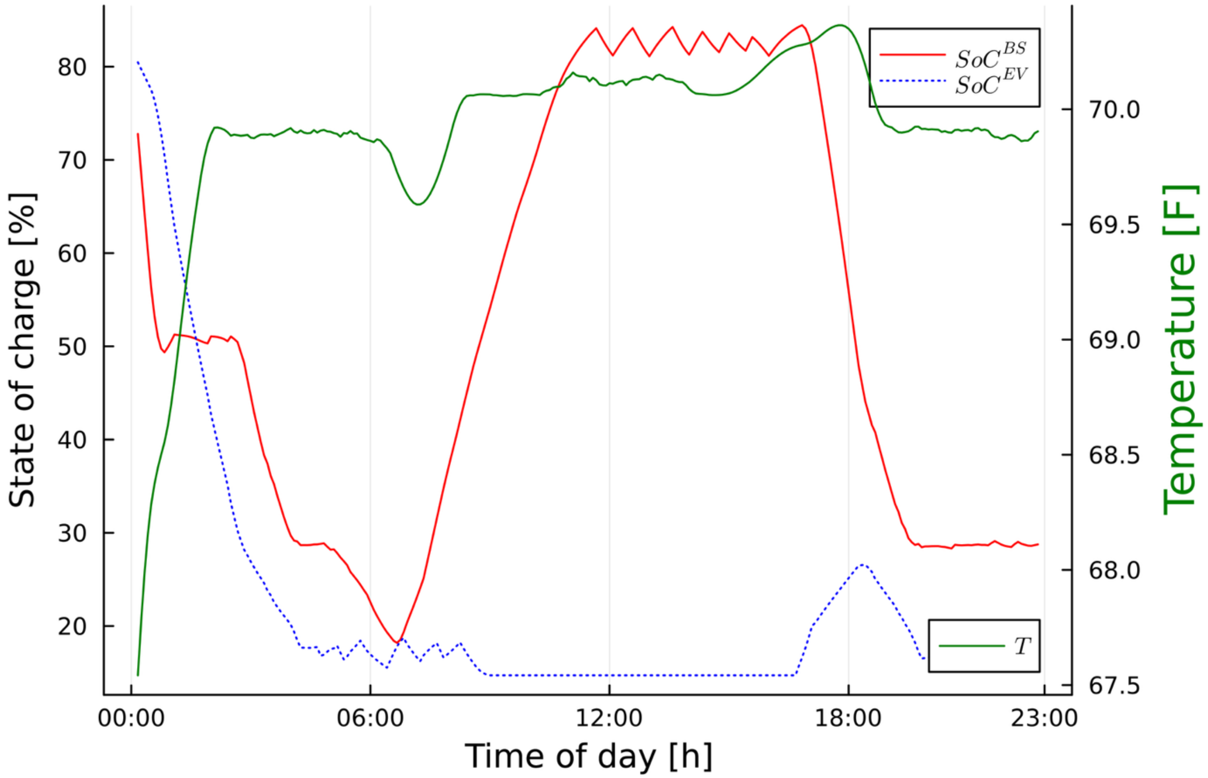
Simulation results on small example CM: Injections

- Simulated 1 CMO & 3 CMAs, using temperature/solar data from California & main grid prices from CAISO
- CMO remains net load throughout the day, with net load lower mid-day during peak solar PV output
- Grid operator requests varying amounts of load flexibility or demand response throughout the day
- CMO is able to successfully aggregate flexibilities of its CMAs to satisfy the regulation signal from the grid operator



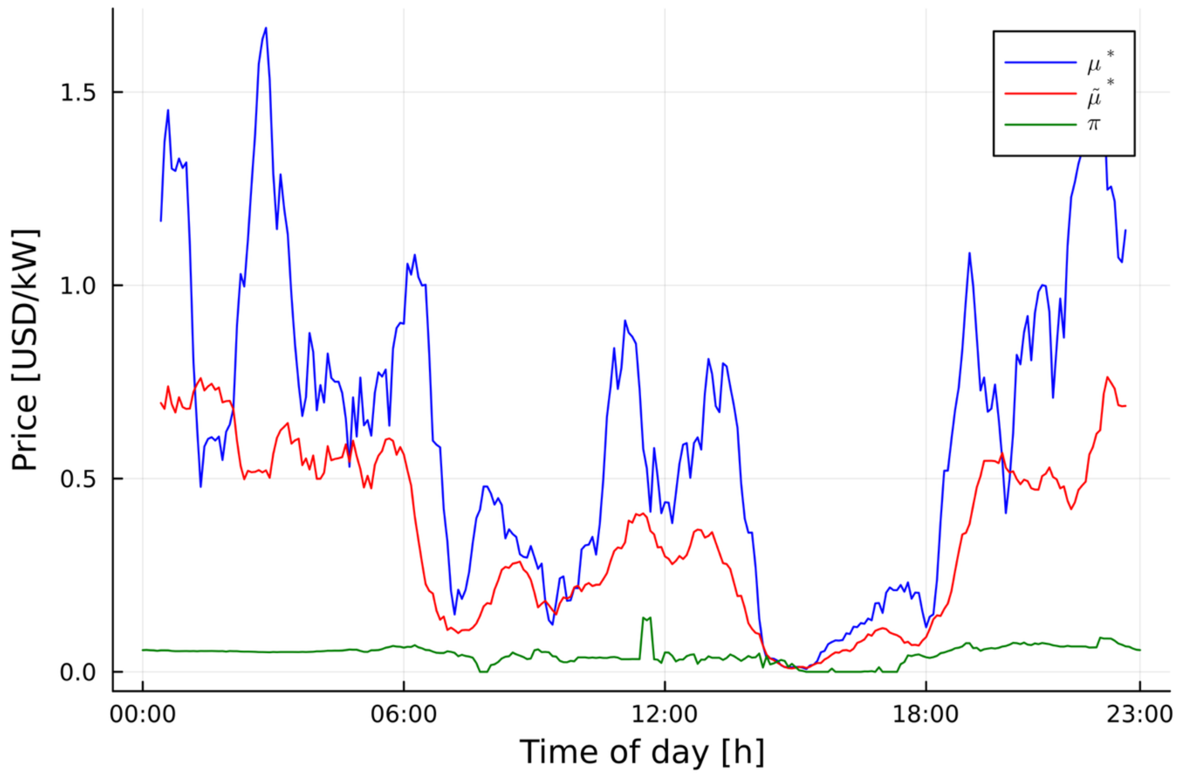
DER coordination for CMA 1





Electricity & flexibility prices

- CM prices are about an order of magnitude higher than the grid price π
- For CMO to provide required flexibility, it has to increase its prices & also compensate CMAs and DERs sufficiently, which raises costs
- Possible approach to mitigate price impacts: Varying prices $\mu_i^*, \tilde{\mu}_i$ for each CMA i instead of common CM rate
  - Pros: More efficient, equitable, fair tariffs
  - Cons: Makes equilibrium analysis more challenging



Conclusions

- Game-theoretic market mechanism for DER flexibility
- Hierarchical approach: Satisfy all DER physical constraints
- Market operator sets strategy-proof optimal prices for both electricity & flexibility
- Pricing induces truthful flexibility bids from all agents
- Meet total required flexibility (net load curtailment)
- Established unique equilibria among market participants with closed form analytical solutions
- <u>Future work</u>: Relax model assumptions, explore other types of equilibria