Circuit-Aware Distributed Optimal Voltage Control for Distribution Grids

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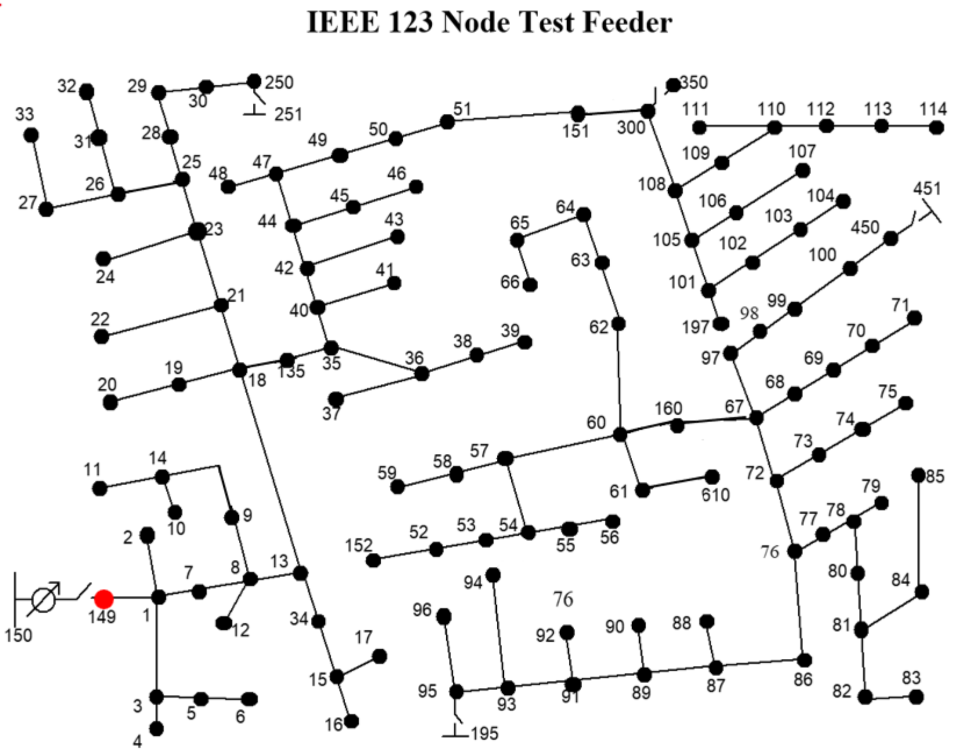
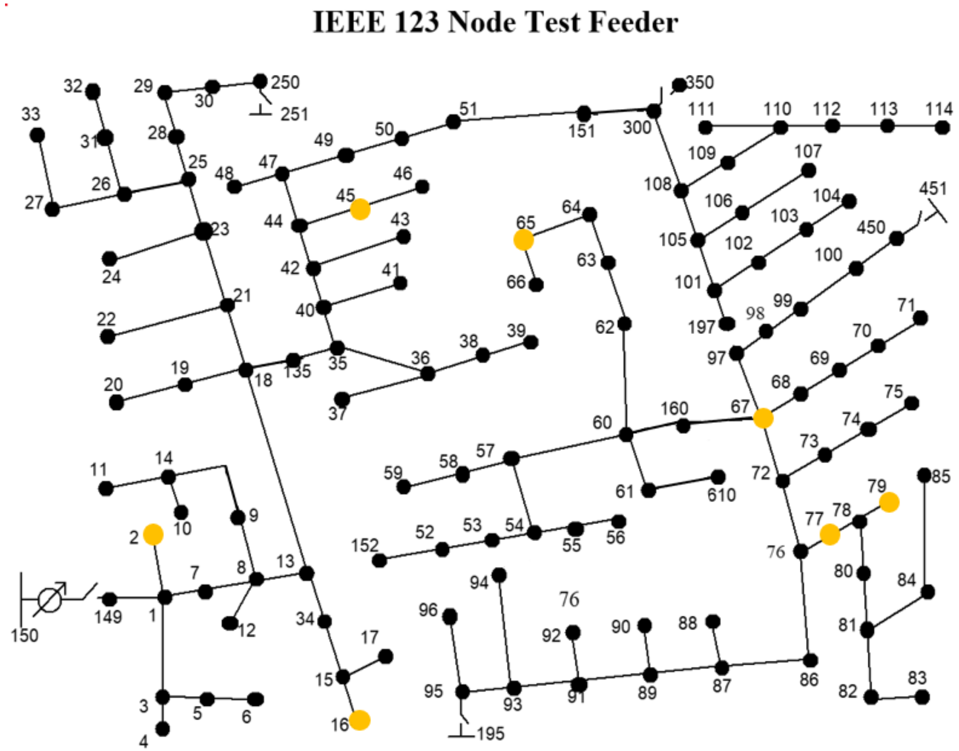
Traditional Distribution Grids



Transmission grid

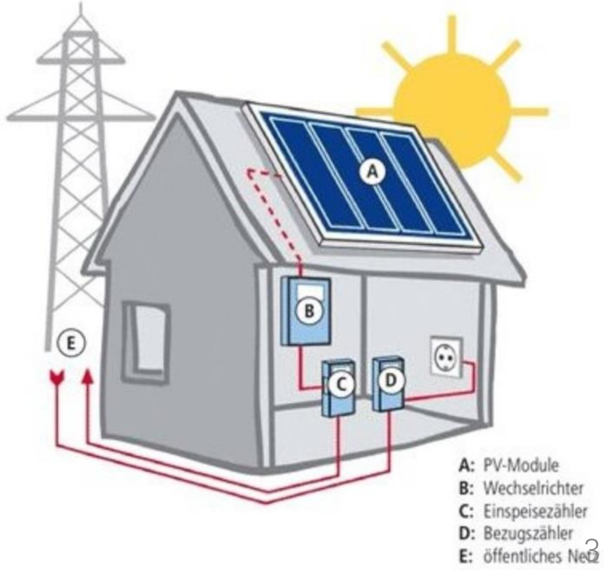
- Connected to transmission grid
- Centralized power generation
- Synchronous generators (SGs) have high inertia & natural time delay
  → Slower dynamics than grid circuits

Future distribution grids



● Distributed energy resources (IBRs)

- More independent operation
- Multiple small inverter-based resources (IBRs) based on power electronics (PE)
  - Solar PV, wind or batteries
- PE dynamics are faster than SGs
  → Little to no inertia or delay



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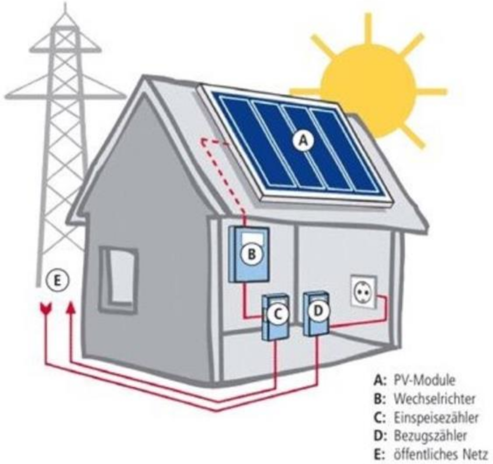
Fast PE Dynamics

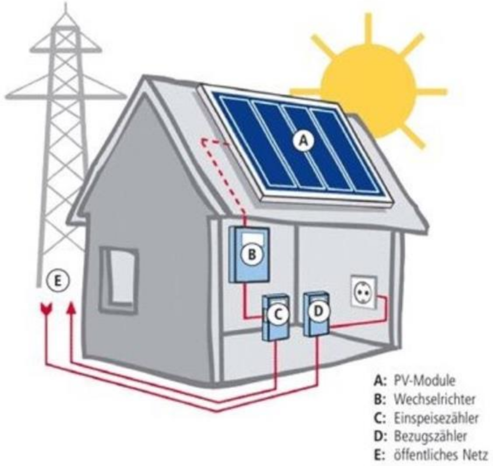


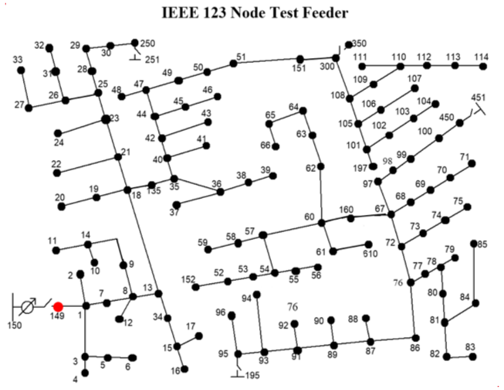
- PE dynamics operate faster than power grid circuits
- **Prior works:** Slow down PE dynamics with virtual inertia or low-pass filter-based droop controllers

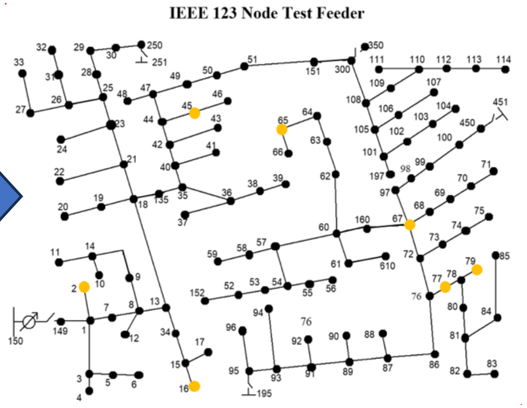
→ Requires large, expensive energy storage

- **Alternative:** Allow fast PE dynamics
  → Need to model fast circuit dynamics too

Challenges for the Future Grid







● Distributed energy resources (IBRs)

Dominant fast circuit dynamics:

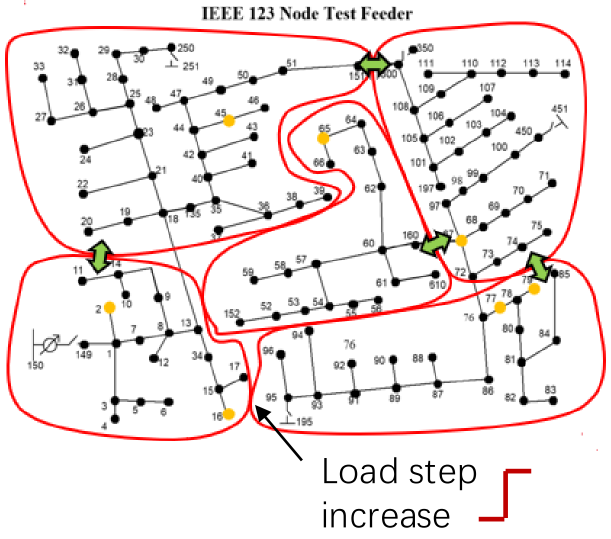
→ Not currently considered in control design

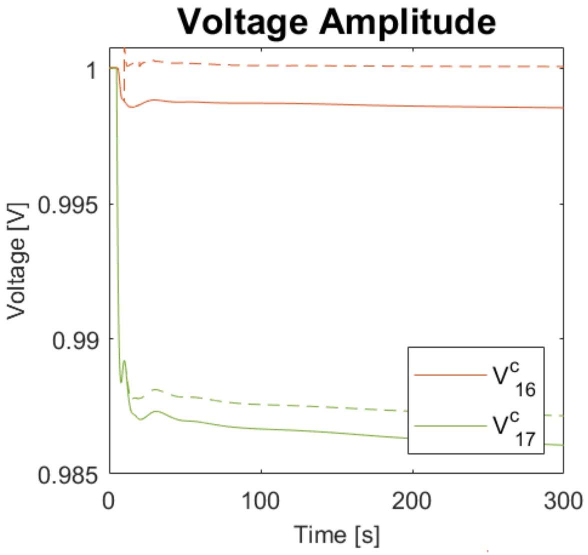
Complex interactions between many IBRs:

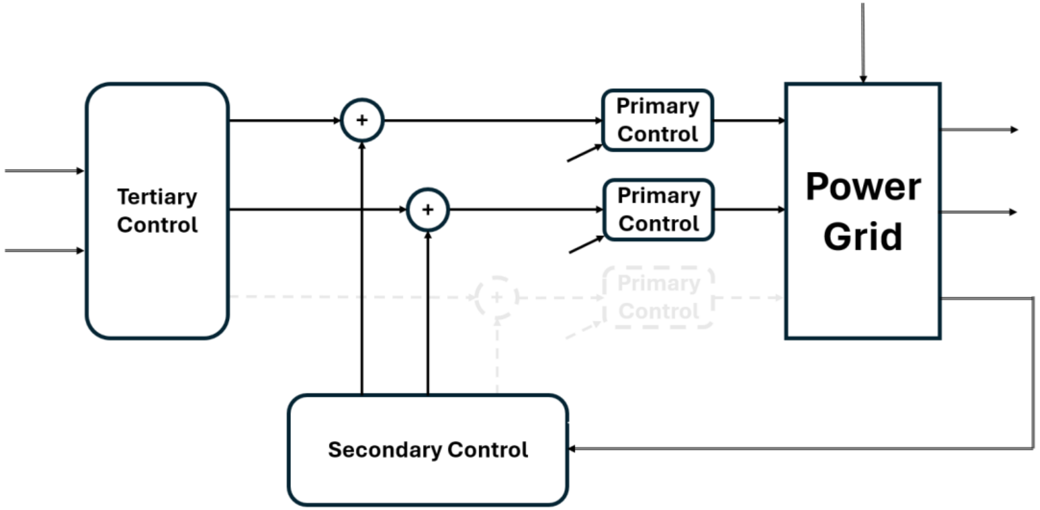
→ Oscillations and instabilities

→ Inefficient power dispatch

Our Work







Hierarchical voltage control

1. Tertiary control via market
2. Model predictive control for secondary control with circuit dynamics model
   - Reduced voltage oscillations
   - Higher efficiency
3. Distributed solver on clustered distribution grid
   - Modular, scalable computation
   - Local updates for local changes

Scalable simulation of multi-IBR power grid with circuits

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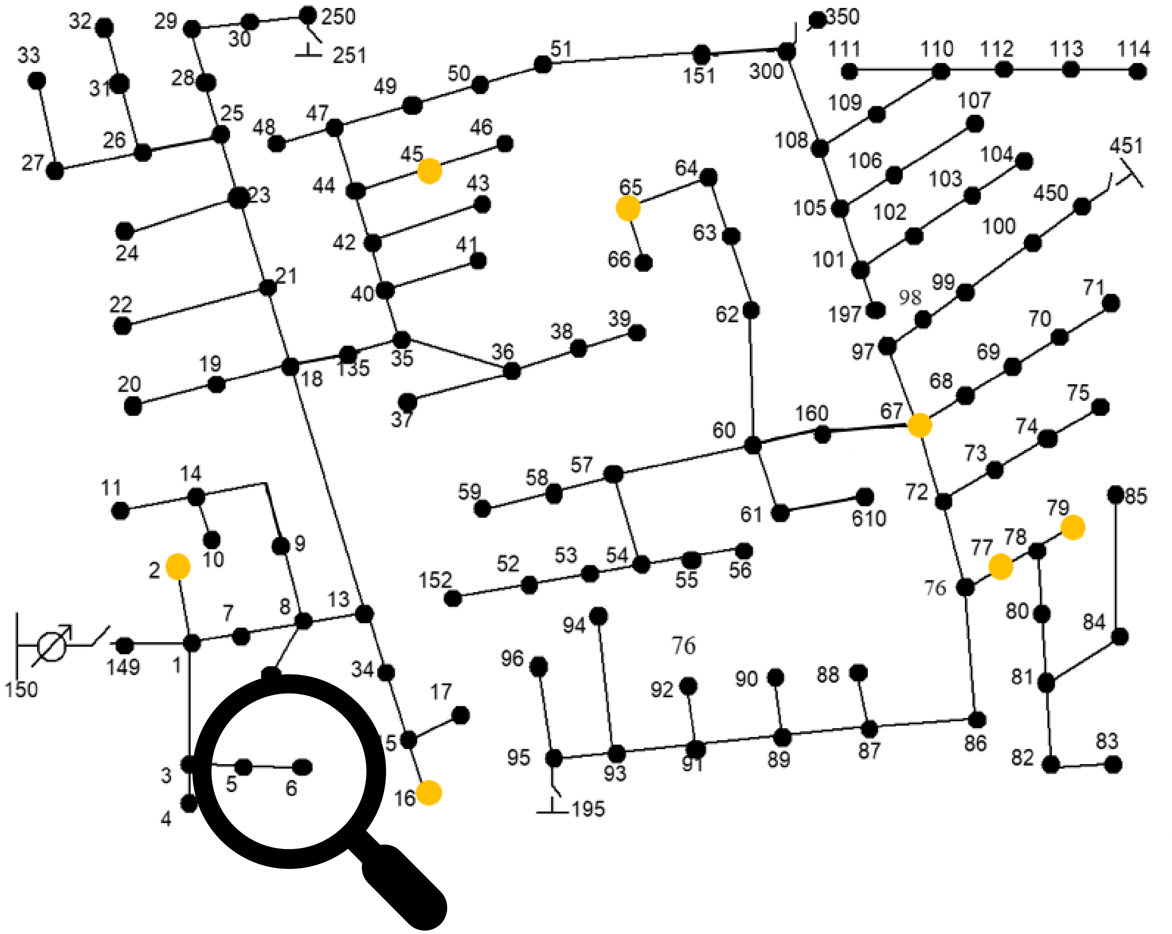
Passive Circuits

IEEE 123 Node Test Feeder



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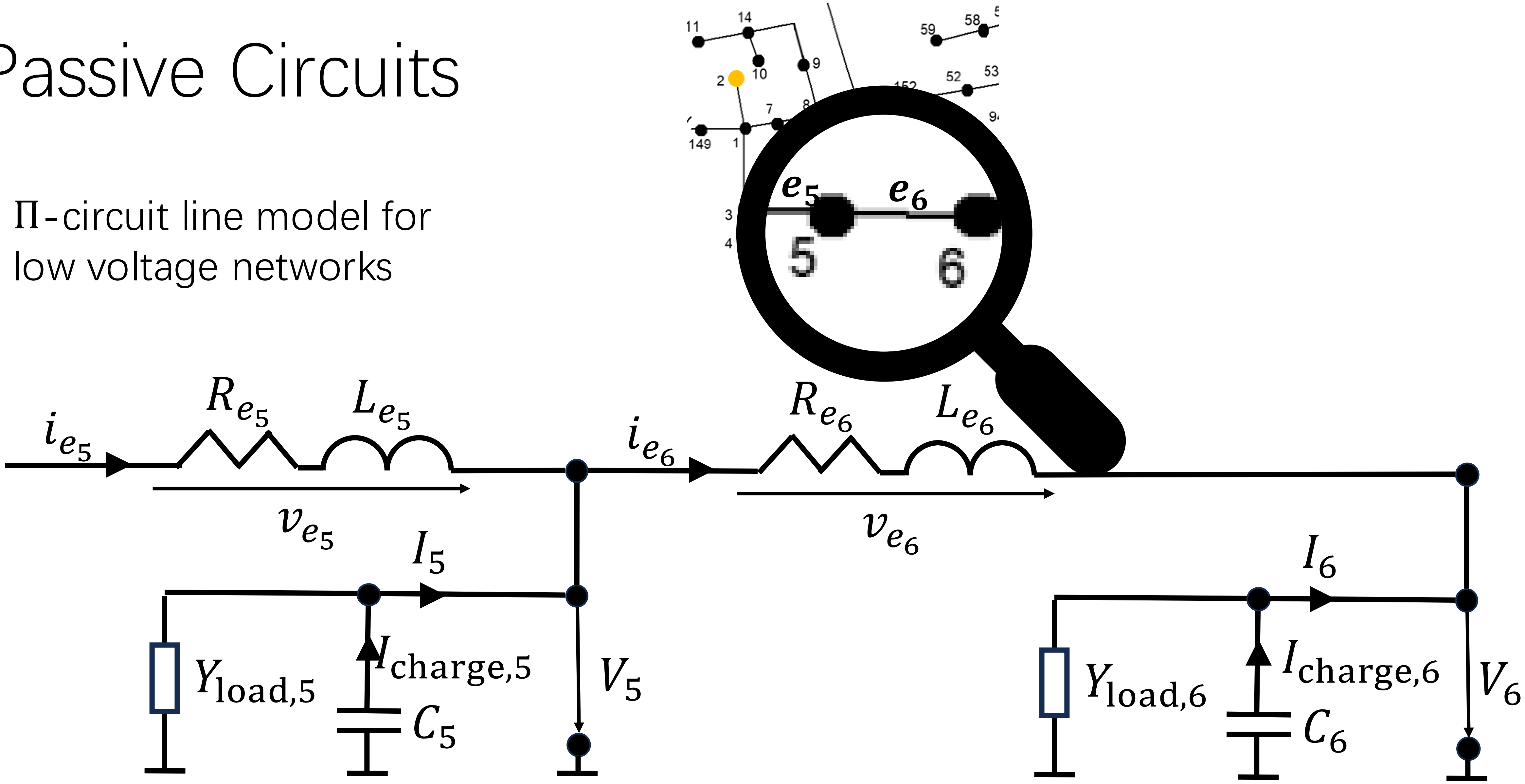
Passive Circuits

Π-circuit line model for low voltage networks



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Passive Circuit Dynamics

Dynamics of power line $\boldsymbol{e}$

$$\boldsymbol{L}_e \frac{d}{dt}\boldsymbol{i}_e(t) = -\boldsymbol{R}_e \boldsymbol{i}_e(t) + v_e(t)$$

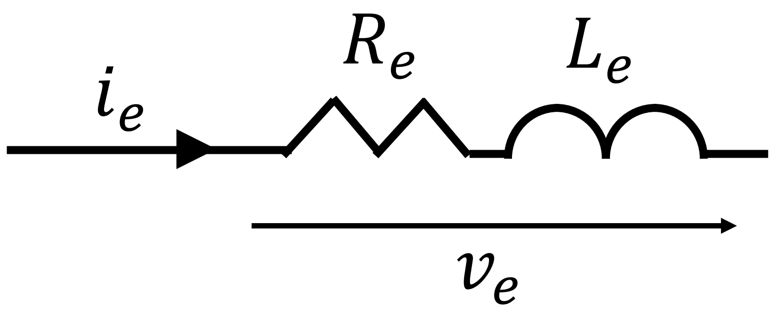
Dynamics of Bus $\boldsymbol{n}$ with resistive load

$$\boldsymbol{C}_n \frac{d}{dt}\boldsymbol{V}_n(t) = -\boldsymbol{I}_{\text{charge},n}(t)$$
$$= -\boldsymbol{I}_n(t) - \boldsymbol{Y}_{\text{load},n}\boldsymbol{V}_n(t)$$

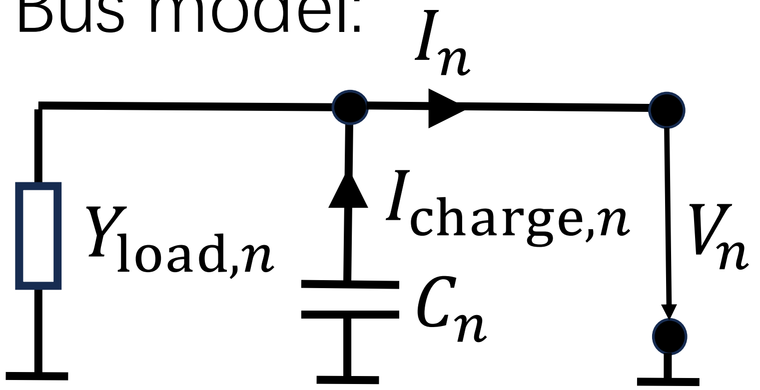
Passive grid dynamics

$$\begin{bmatrix} \boldsymbol{L} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{C} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} \boldsymbol{i} \\ \boldsymbol{V} \end{bmatrix} = \begin{bmatrix} -\boldsymbol{Z} & \boldsymbol{B}^\top \\ -\boldsymbol{B} & -\boldsymbol{Y}_{\text{load}} \end{bmatrix} \begin{bmatrix} \boldsymbol{i} \\ \boldsymbol{V} \end{bmatrix}$$

Power line model:



Bus model:

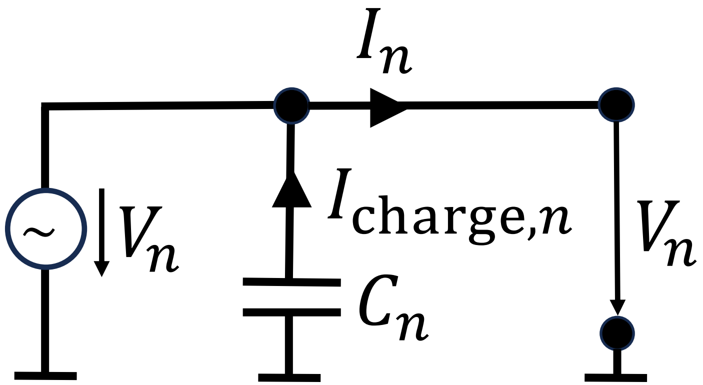


Florian Dörfler, Dominic Groß, „Control of Low-Inertia Power Systems,"
Annual Review of Control, Robotics, and Autonomous Systems, Volume 6, 2023.

Primary Control

Generator is controlled as grid-forming (GFM) converter:

- Assume voltage and current control in steady-state
- Assume ideal behavior within power range



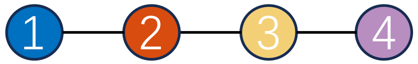
Converter droop control:

$$\frac{d}{dt}\hat{V}_n(t) = -k_{q,n}\Delta q_n(t)$$

Full primary grid dynamics:

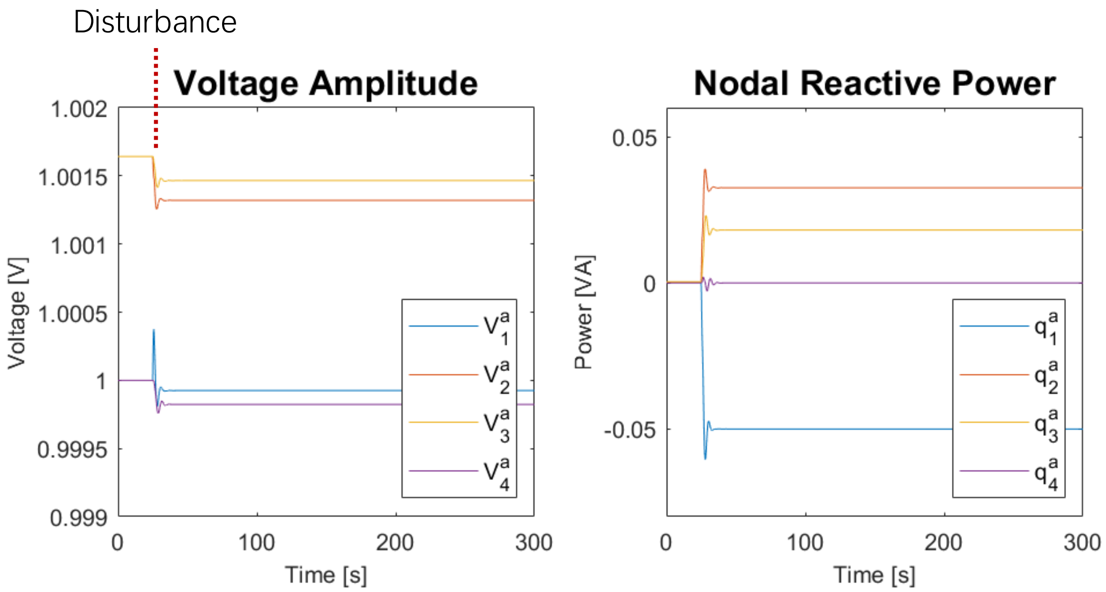
$$\begin{bmatrix} \boldsymbol{L} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{C} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} \boldsymbol{i}(t) \\ \boldsymbol{V}(t) \end{bmatrix} = \begin{bmatrix} -\boldsymbol{Z} & \boldsymbol{B}^\top \\ -\bar{\boldsymbol{G}}\boldsymbol{B} & -\bar{\boldsymbol{G}}\boldsymbol{Y}_{\text{load}} \end{bmatrix} \begin{bmatrix} \boldsymbol{i}(t) \\ \boldsymbol{V}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ -\boldsymbol{C}\boldsymbol{K}_q\Delta \boldsymbol{q}(t) \end{bmatrix}$$

Secondary control



$P =$ -.5 .5 .5 -.5

$\Delta S_{\text{load}} =$ -.1-.05i 0 0 0



- Regulation of voltage by changing reactive power reference

$$\boldsymbol{q}^{\text{ref}}(k) = \boldsymbol{q}^0 + f_{\text{sec}}(\boldsymbol{V}(k))$$

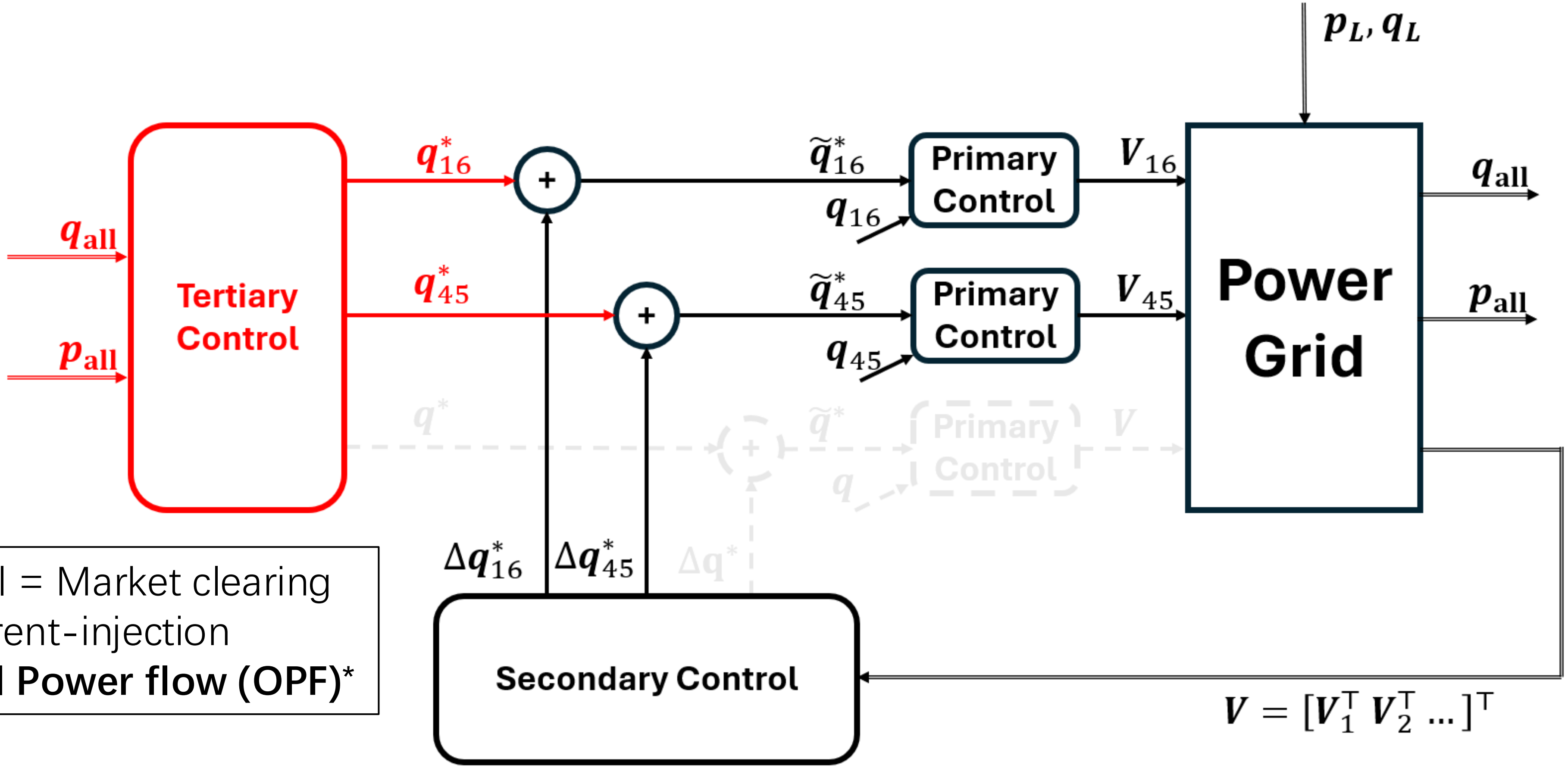
- Traditional implementation with averaging proportional integral (PI) control
  → Apply the same reactive power setpoint change to all IBRs

$$y(s) = (k_p + \frac{k_I}{s})u(s).$$

$$u = \frac{1}{N}\sum_{n\in\mathcal{V}} \Delta\hat{\boldsymbol{V}}_n.$$

$$\Delta q_n^{\text{ref}} = \frac{1}{|V_{\text{gen}}|}y, \ \forall n \in \mathcal{V}_{\text{gen}}$$

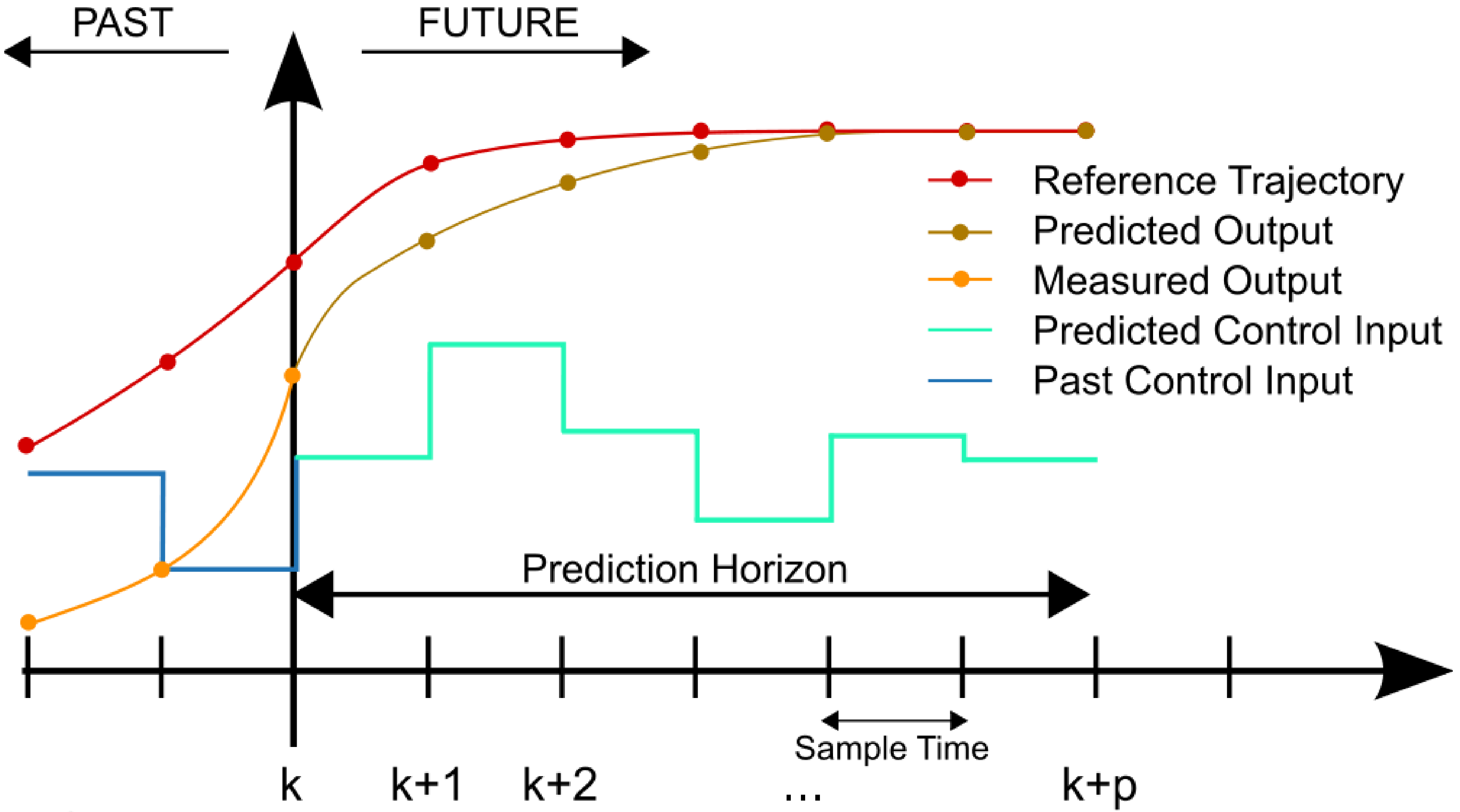
Control Hierarchy



Tertiary control = Market clearing by solving Current-injection based **Optimal Power flow (OPF)***

*A. Potter, et at., A reactive power market for the future grid, Advances in Applied Energy, Volume 9, 2023.

Model Predictive Control



Secondary Control – Why Model Predictive Control?

- Intuitive objective, e.g., voltage tracking and loss reduction

$$\sum_{\ell=k}^{k+N_p-1} ||\boldsymbol{V}^{\mathrm{Re,ref}}(\ell+1)-\boldsymbol{V}^{\mathrm{Re}}(\ell+1)||_2^2+||\boldsymbol{V}^{\mathrm{Im,ref}}(\ell+1)-\boldsymbol{V}^{\mathrm{Im}}(\ell+1)||_2^2+\nu_q||\Delta\boldsymbol{q}^{\mathrm{ref}}(\ell)||_1$$

- Can better plan ahead for bounds (e.g. generator limits)
- More degrees of freedom for multi-IBR settings
- Reduced losses by more local actuation
- Less parameter tuning needed to achieve good performance
- Faster stable response, less oscillations

MPC – Optimization problem

$$\min_{\Delta \boldsymbol{q}^{\text{ref}}(k),\ldots,\Delta \boldsymbol{q}^{\text{ref}}(k+N_p-1)} \sum_{\ell=k}^{k+N_{\text{p}}-1} ||\boldsymbol{V}^{\text{Re,ref}}(\ell+1) - \boldsymbol{V}^{\text{Re}}(\ell+1)||_2^2 \; \ldots$$

$$+ ||\boldsymbol{V}^{\text{Im,ref}}(\ell+1) - \boldsymbol{V}^{\text{Im}}(\ell+1)||_2^2 \; \ldots$$

$$+ \nu_{\text{q}} ||\Delta \boldsymbol{q}^{\text{ref}}(\ell)||_1$$

$$\text{s.t.} \begin{bmatrix} \boldsymbol{i}(\ell+1) \\ \boldsymbol{V}(\ell+1) \end{bmatrix} = \boldsymbol{A} \begin{bmatrix} \boldsymbol{i}(\ell) \\ \boldsymbol{V}(\ell) \end{bmatrix} + \boldsymbol{M} \Delta \boldsymbol{q}^{\text{ref}}(\ell) + \boldsymbol{c}, \; \forall \ell,$$

Cost function:

- ℓ_2 penalty on voltage
- ℓ_1 penalty on reactive power

Discrete, linearized grid dynamics

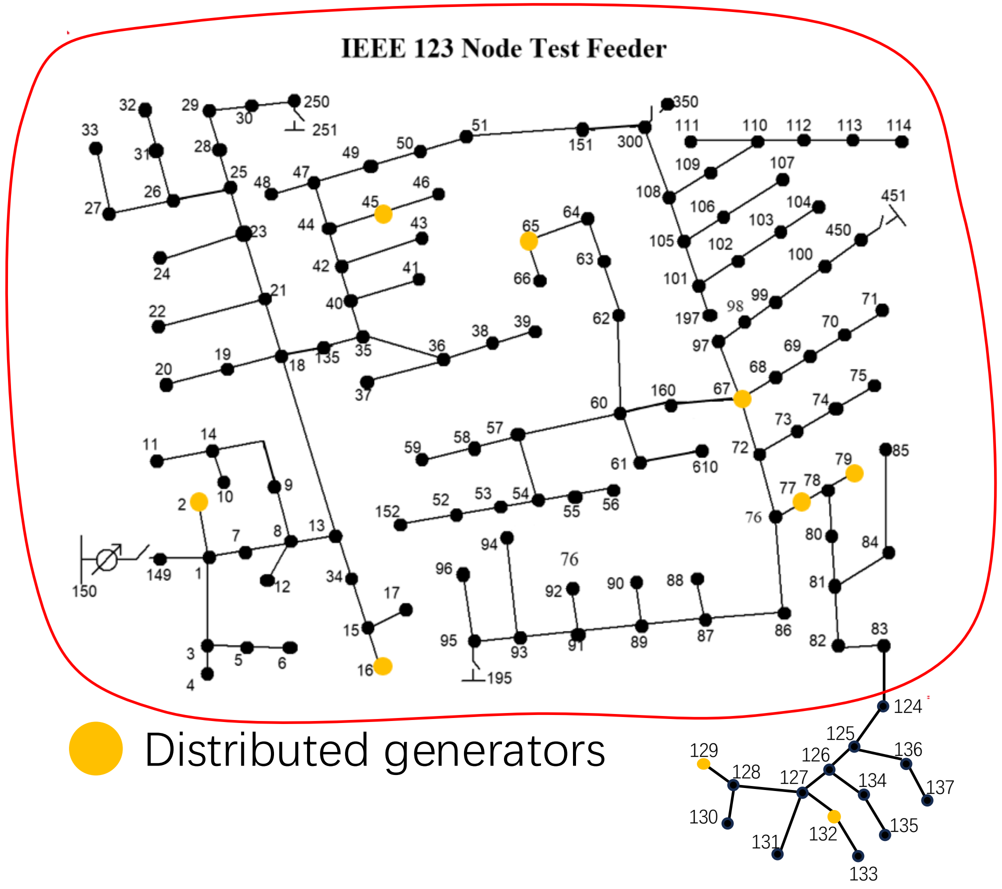
Initial conditions

Voltage bounds

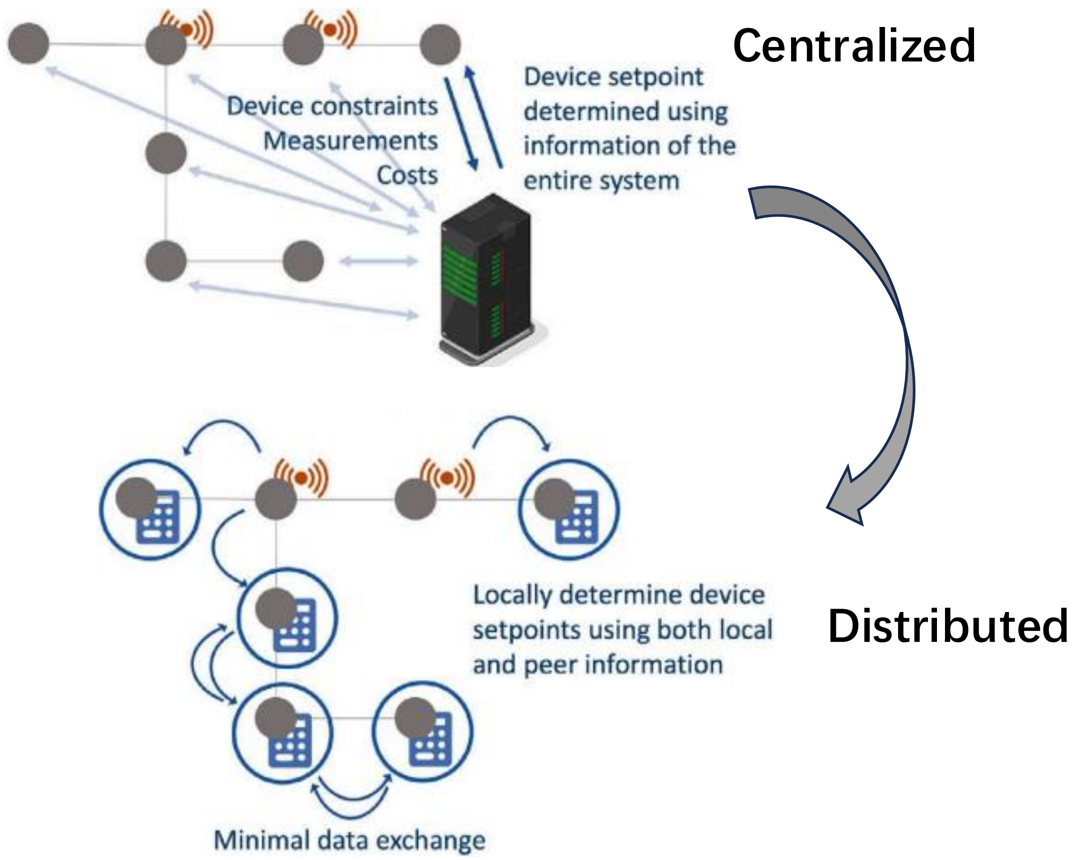
Generation bounds

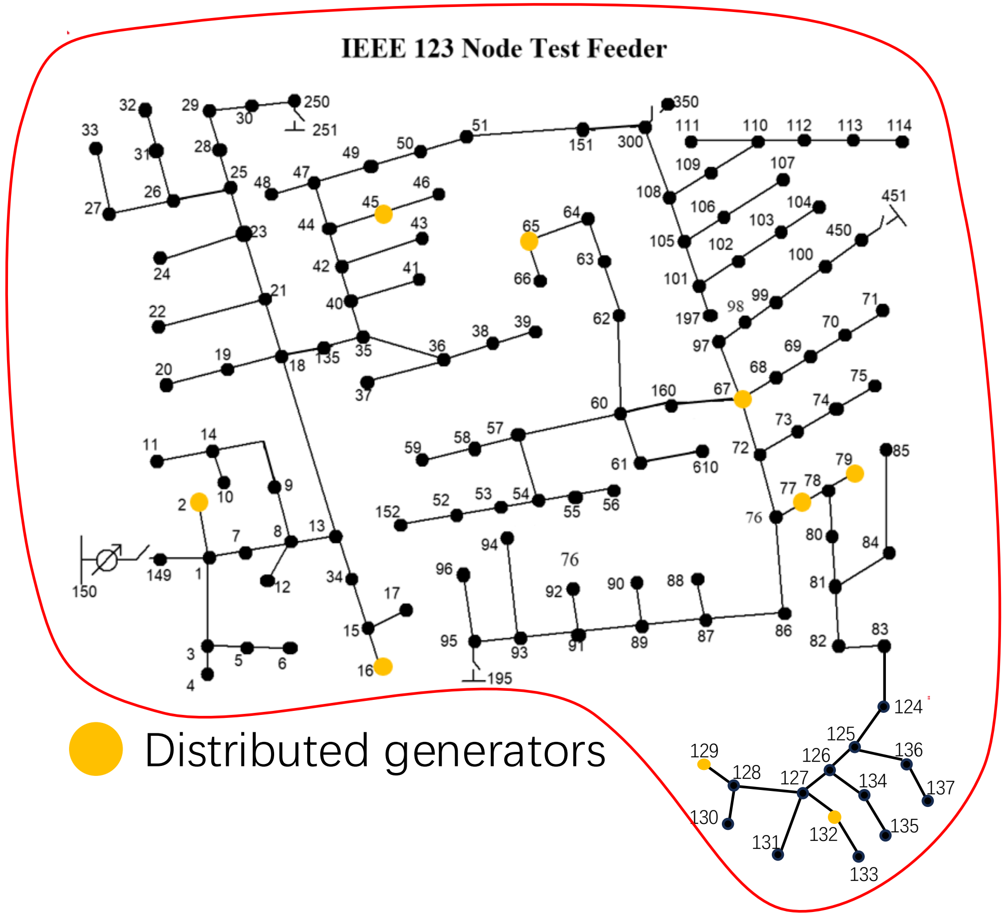
Centralized MPC

Centralized



Centralized MPC





Priyank Srivastava, et at., "Voltage regulation in distribution grids: A survey," Annual Reviews in Control, vol. 55, pp. 165-181, 2023.

Distributed MPC

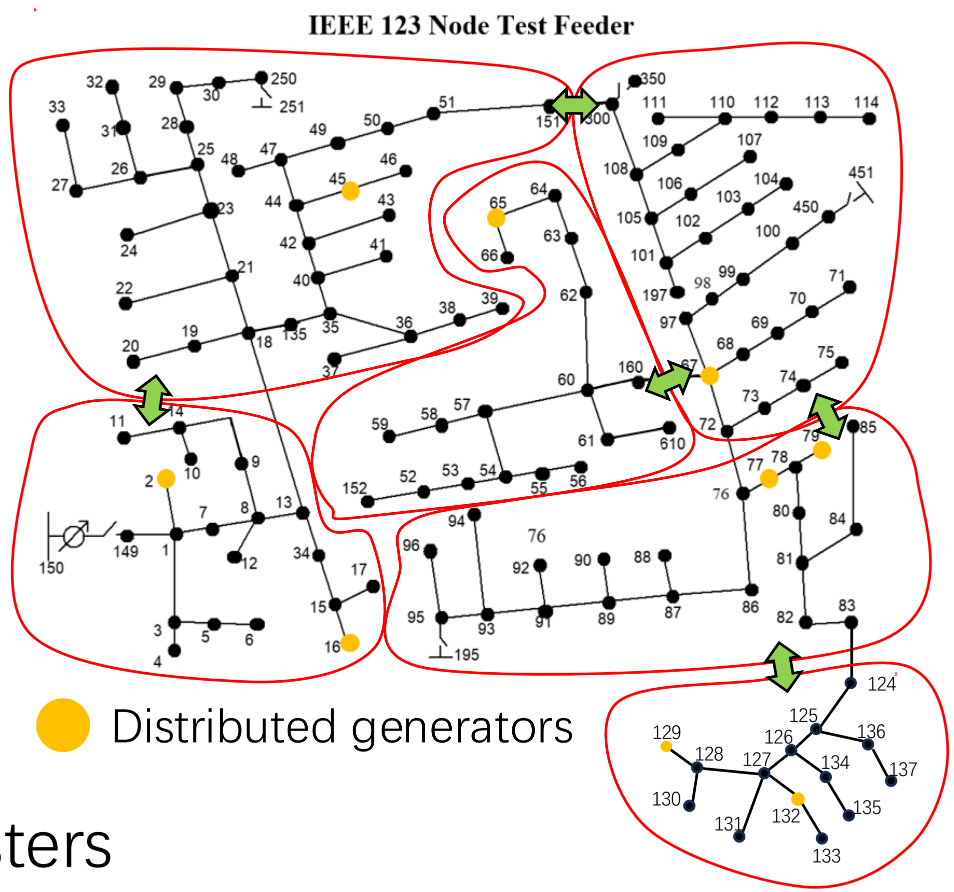
- Separate grid into multiple clusters
- Solve distributed optimization problem

→ **Separation** of computational burden

→ More **resilient** against local failures: No central unit, failures in a cluster don't affect others

→**Privacy**: No exchange of sensitive info (e.g. topology, generation, load) between clusters

→Local changes stay local



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T. Heins, et al., "Distributed Frequency Divider-based Model Predictive Control for Frequency Control of Multi-Area Power Systems," Smart Grid Comm, 2023.

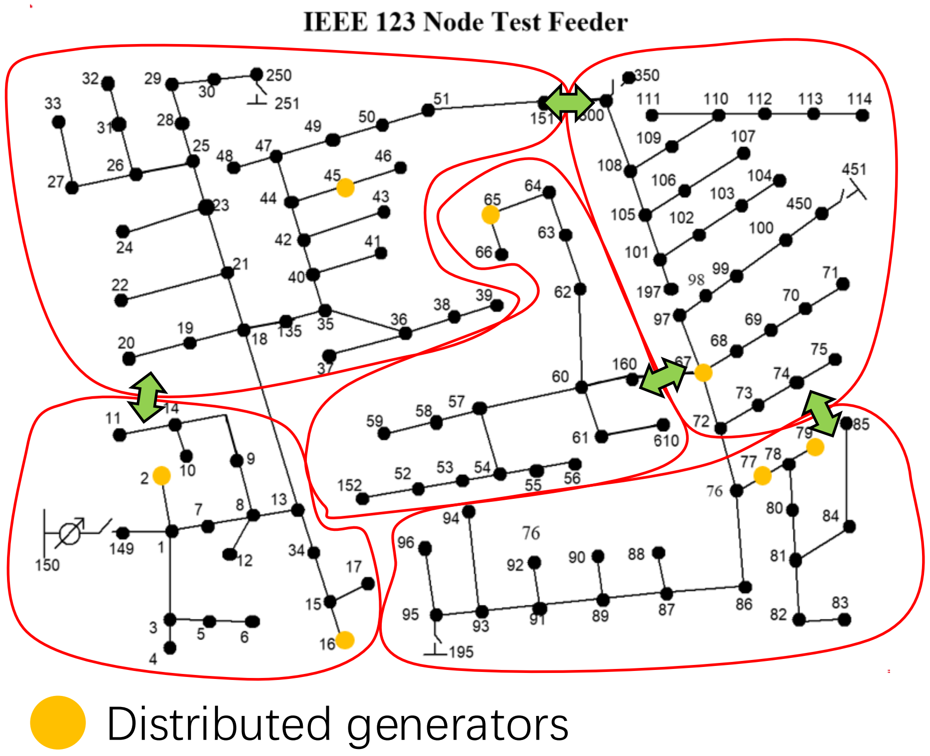
DMPC approach

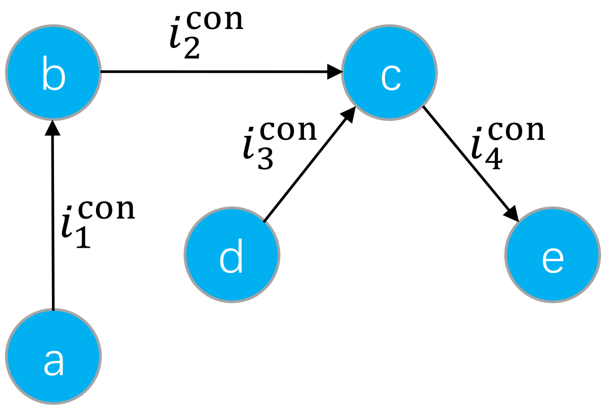
- Derive coupled cluster dynamics
- Define DMPC optimization problem
- Bring distributed optimization into standard separable form for solver

$$\min_{U_c(k)\,\in\,\mathcal{U}_c} \sum_{c\in\mathcal{C}} f_c(U_c(k))$$

$$\text{s.t.} \quad \sum_{c\in\mathcal{C}} \mathcal{A}_c U_c(k) = d$$

- Solve using algorithm based on alternating direction method of multipliers (ADMM)

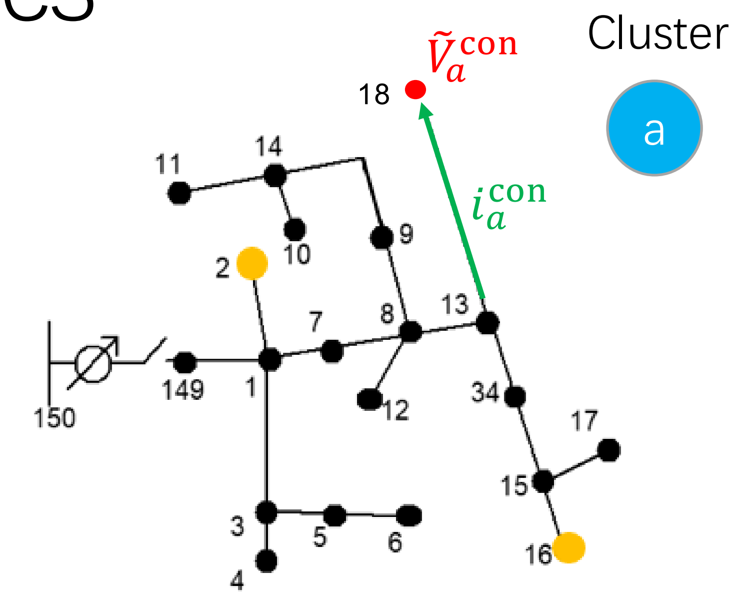




A. Falsone, et al., “Tracking-ADMM for distributed constraint-coupled optimization,” Automatica, Volume 117, 2020.

DMPC – Coupled Cluster Dynamics

- Dynamics within each cluster remain the same
- Clusters exchange subset of variables with neighbors
  - Current i_a^{con} flowing to other cluster (connected at node 18)
- Maintain variable copies to satisfy coupled network constraints
  - Connected voltage $\tilde{V}_a^{\mathrm{con}}$ of other cluster



$$\frac{d}{dt}\begin{bmatrix}\tilde{x}_a(t)\\ i_a^{\mathrm{con}}(t)\end{bmatrix}=\begin{bmatrix}\tilde{A}_a & \tilde{A}_a^{\mathrm{conx}}\\ \tilde{A}_a^{\mathrm{coni}} & -\tilde{R}_a^{\mathrm{con}}\end{bmatrix}\underbrace{\begin{bmatrix}\tilde{x}_a(t)\\ i_a^{\mathrm{con}}(t)\end{bmatrix}}+\begin{bmatrix}\tilde{M}_a & 0\\ 0 & \tilde{B}_a^{\mathrm{con}}\end{bmatrix}\underbrace{\begin{bmatrix}\Delta q_a^{\mathrm{ref}}(t)\\ \tilde{V}_a^{\mathrm{con}}(t)\end{bmatrix}}+\begin{bmatrix}\tilde{c}_a\\ 0\end{bmatrix},$$

$$\underbrace{\tilde{V}_a^{\mathrm{con}}(t)=V_{18}(t).}$$

Internal cluster state: $\tilde{x}_a$
External cluster state: i_a^{con}

Internal cluster input: $\Delta q_a^{\mathrm{ref}}$

Ensures that true voltage matches copied value

DMPC – Optimization Problem

$$\min_{\Delta q^{\mathrm{ref}}(k),\ldots,\Delta q^{\mathrm{ref}}(k+N_p-1)} \sum_{c\in\mathcal{C}} \sum_{\ell=k}^{k+N_{\mathrm{p}}-1} ||\boldsymbol{V}_c^{\mathrm{Re,ref}}(\ell+1) - \boldsymbol{V}_c^{\mathrm{Re}}(\ell+1)||_2^2 \ \ldots$$

$$+ ||\boldsymbol{V}_c^{\mathrm{Im,ref}}(\ell+1) - \boldsymbol{V}_c^{\mathrm{Im}}(\ell+1)||_2^2 \ \ldots$$

$$+ \nu_{\mathrm{q}}||\Delta \boldsymbol{q}_c^{\mathrm{ref}}(\ell)||_1$$

Cost function separated by clusters

$$\text{s.t.} \quad \boldsymbol{x}_c(\ell+1) = \boldsymbol{A}_c\boldsymbol{x}_c(\ell) + \boldsymbol{M}_c\boldsymbol{u}_c(\ell) + \boldsymbol{c}_c\forall\ell,\ \forall c \in \mathcal{C},$$

Cluster dynamics

$$\tilde{\boldsymbol{v}}_{c_i}^{\mathrm{con}}(\ell) = \tilde{\boldsymbol{B}}_{c_j,c_i}^{\mathrm{con}}\tilde{\boldsymbol{x}}_{c_j}(\ell),\ \forall(c_i,c_j)\in\mathcal{E}_{\mathrm{c}},\ \forall\ell,$$

$$\tilde{\boldsymbol{v}}_{c_j}^{\mathrm{con}}(\ell) = \tilde{\boldsymbol{B}}_{c_i,c_j}^{\mathrm{con}}\tilde{\boldsymbol{x}}_{c_i}(\ell),\ \forall(c_i,c_j)\in\mathcal{E}_{\mathrm{c}},\ \forall\ell,$$

Consensus constraints for voltage copies

$$\boldsymbol{i}(k) = \boldsymbol{i}_0,$$

$$\boldsymbol{V}(k) = \boldsymbol{V}_0,$$

Initial conditions

$$\underline{\boldsymbol{V}}^{\mathrm{Re}} \le \boldsymbol{V}(\ell)^{\mathrm{Re}} \le \bar{\boldsymbol{V}}^{\mathrm{Re}},\ \forall\ell,$$

$$\underline{\boldsymbol{V}}^{\mathrm{Im}} \le \boldsymbol{V}(\ell)^{\mathrm{Im}} \le \bar{\boldsymbol{V}}^{\mathrm{Im}},\ \forall\ell,$$

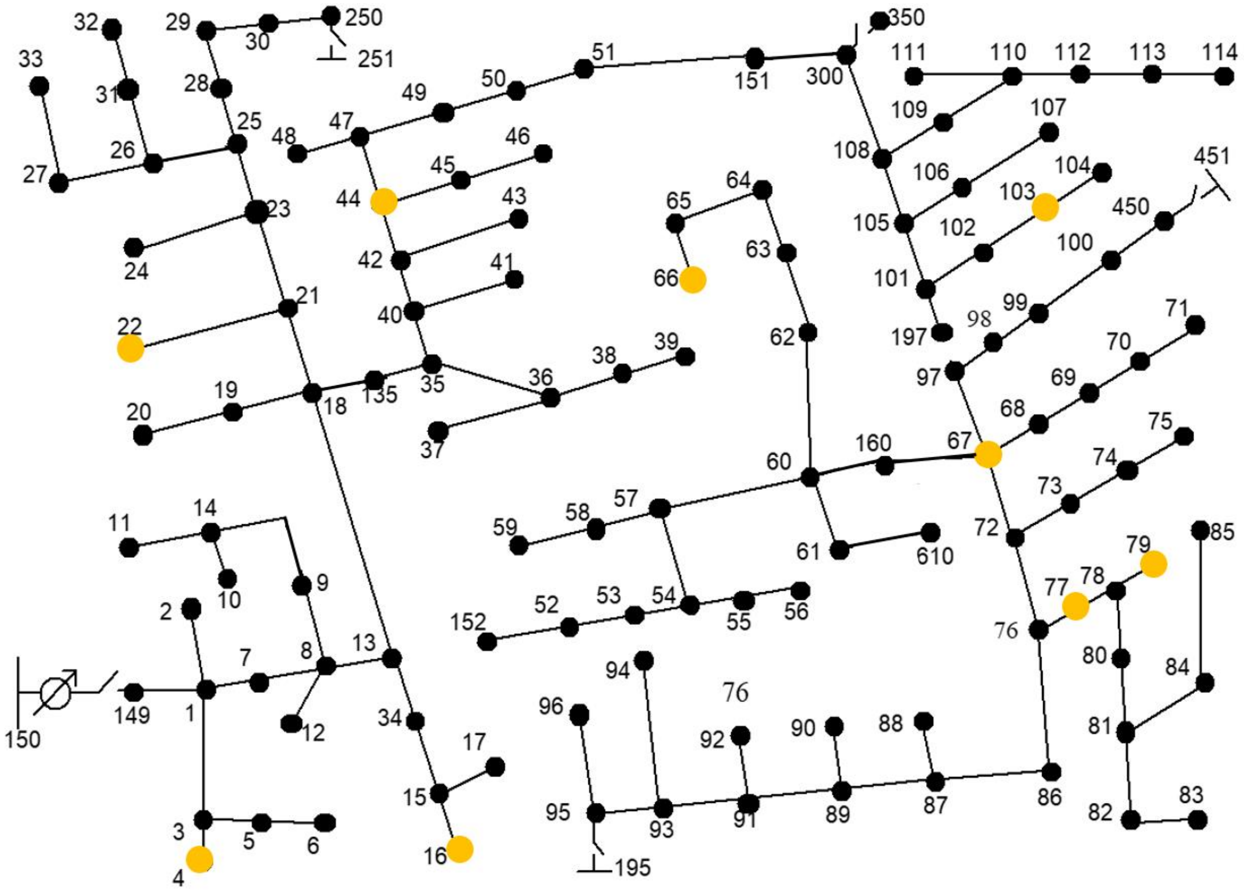
Voltage bounds

$$\Delta\underline{\boldsymbol{q}}^{\mathrm{ref}} \le \Delta\boldsymbol{q}^{\mathrm{ref}}(\ell) \le \Delta\bar{\boldsymbol{q}}^{\mathrm{ref}},\ \forall\ell,$$

Generation bounds

Simulation on realistic feeder

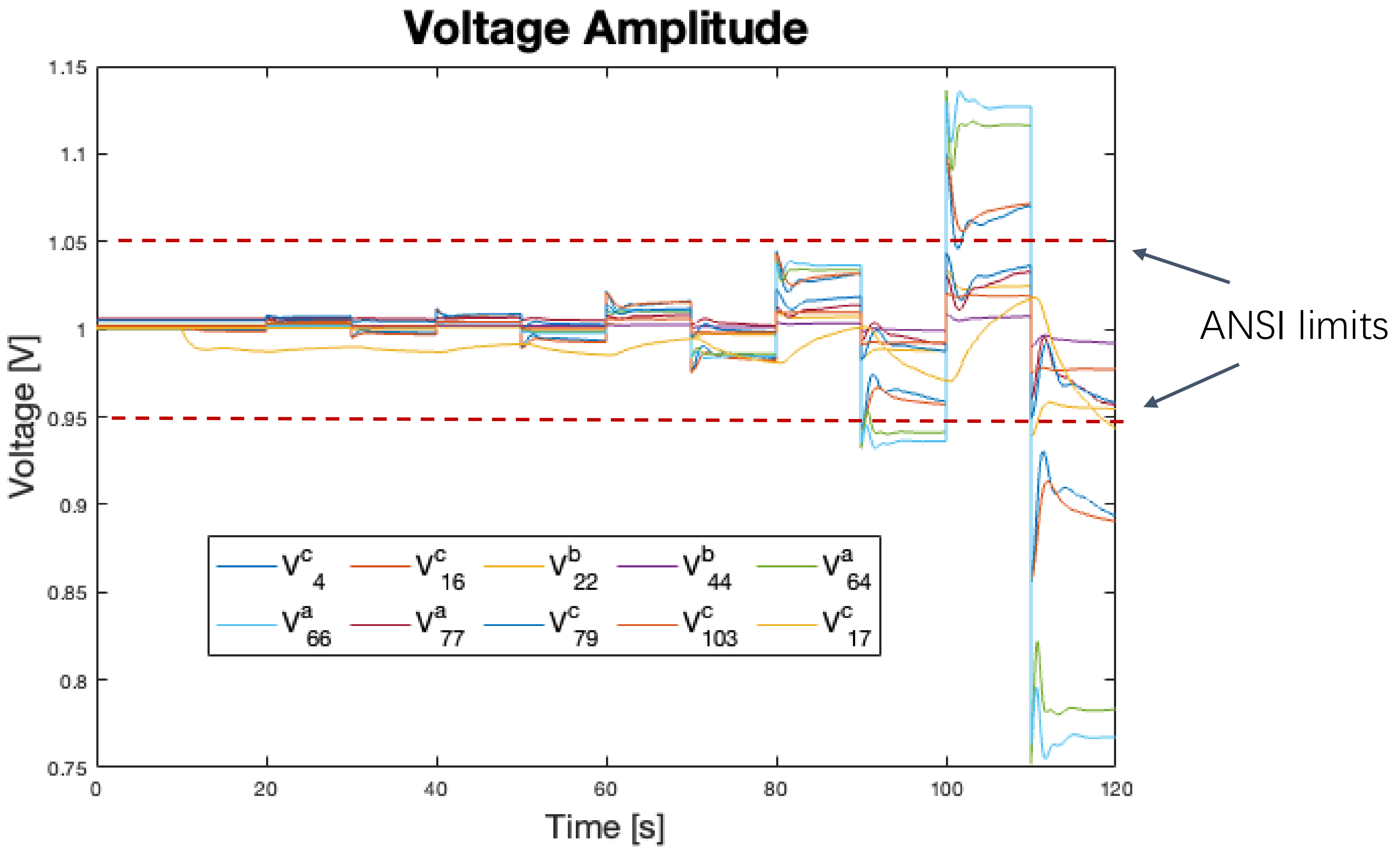
IEEE 123 Node Test Feeder



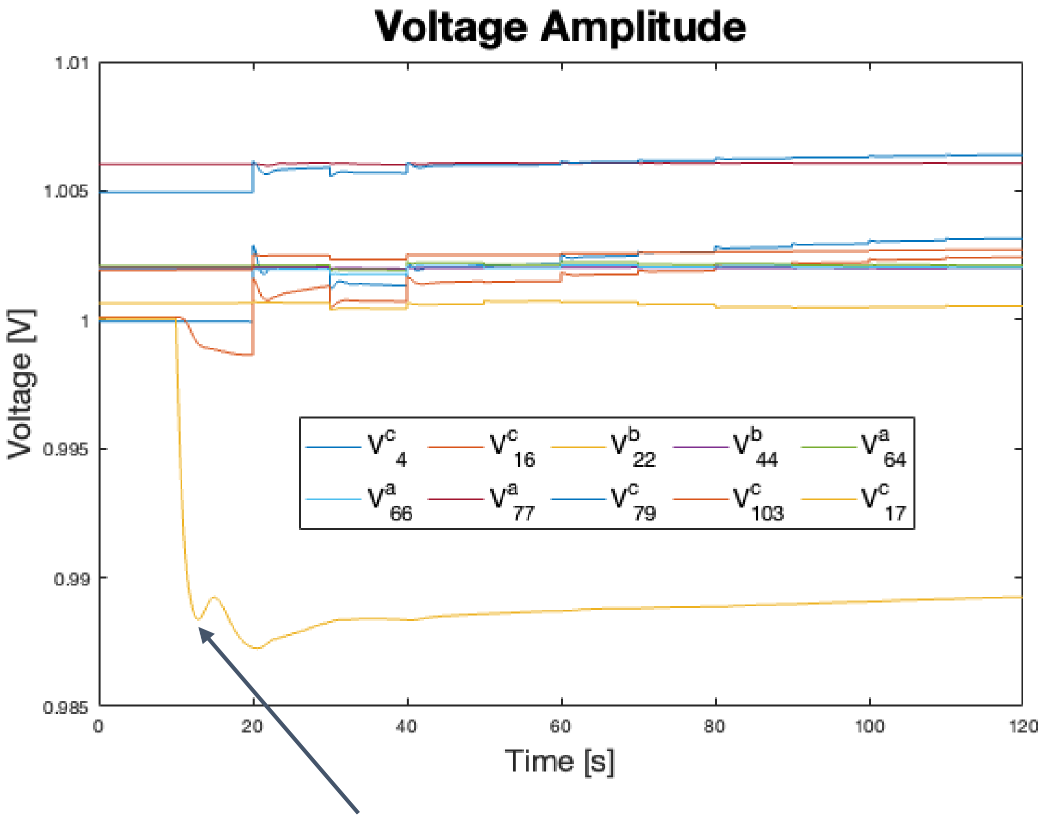
Generators

- 9 large IBRs distributed throughout network
- Simulate network as microgrid → No power from main transmission grid
- Introduce large sudden increase in load at node 17

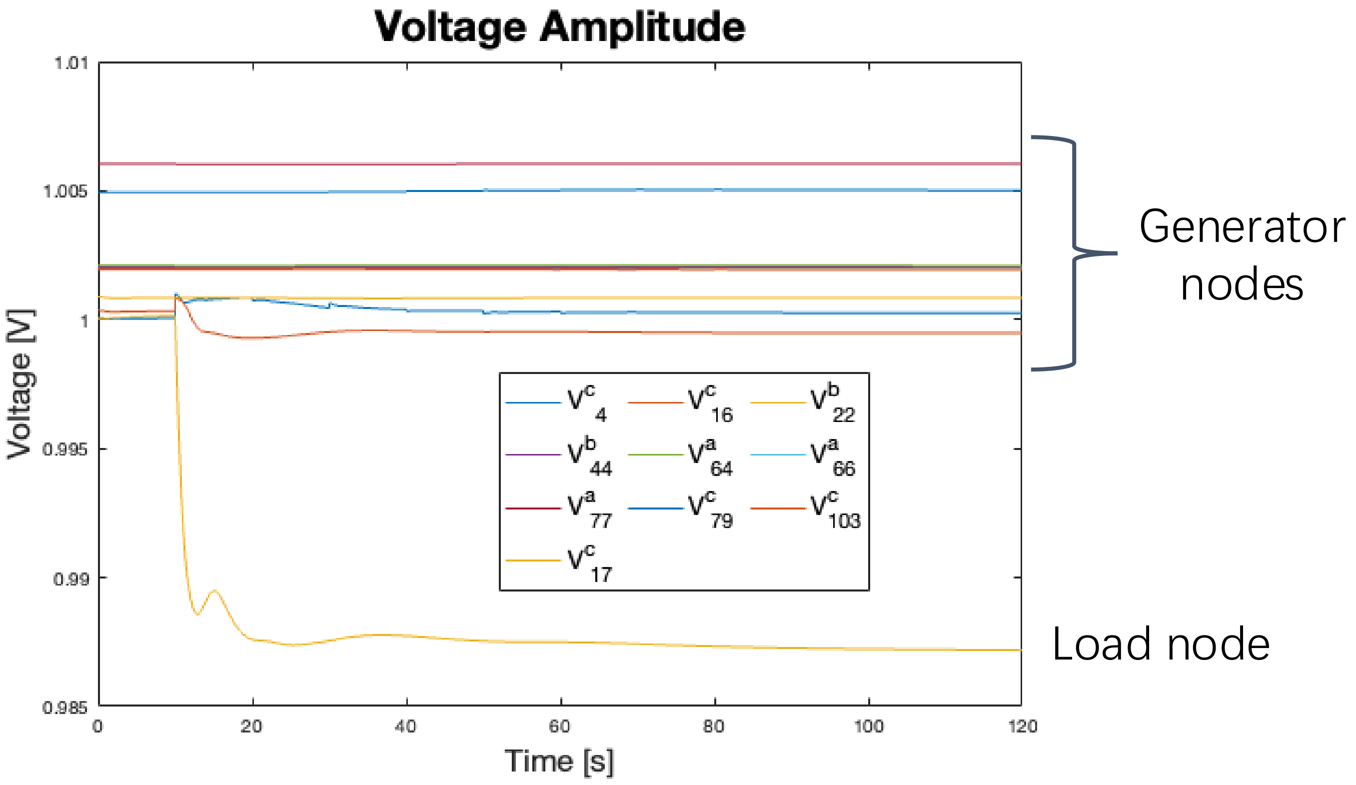
PI control → Can be unstable without tuning



PI control



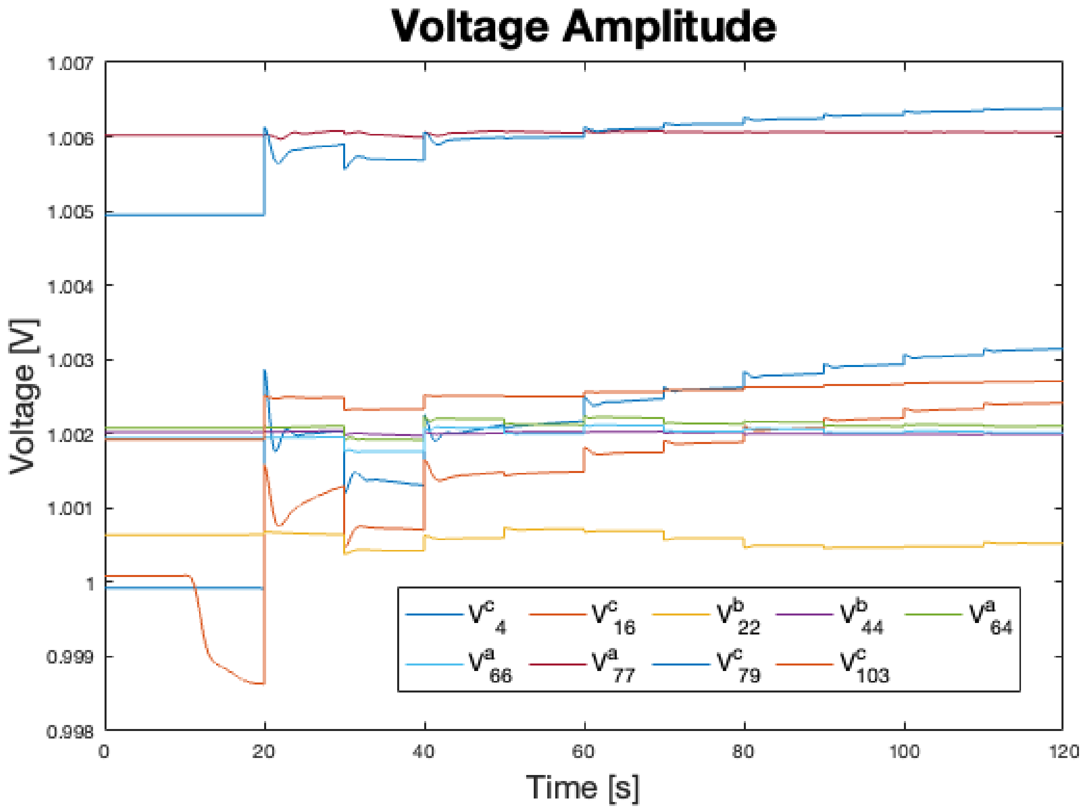
MPC



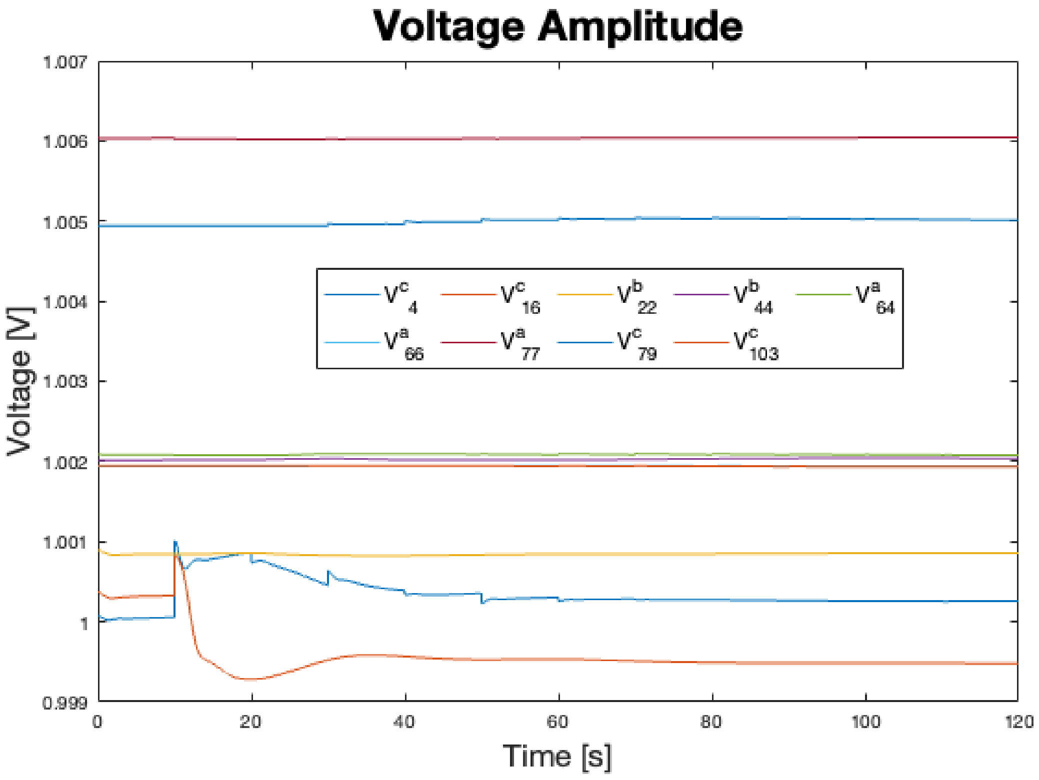
Faster stable response
with almost no oscillations

Zooming into only generator nodes

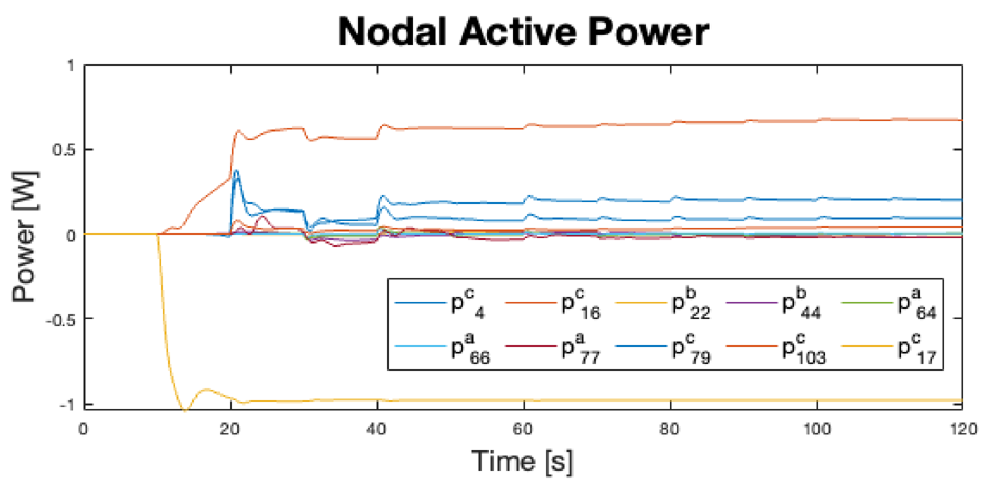
PI control

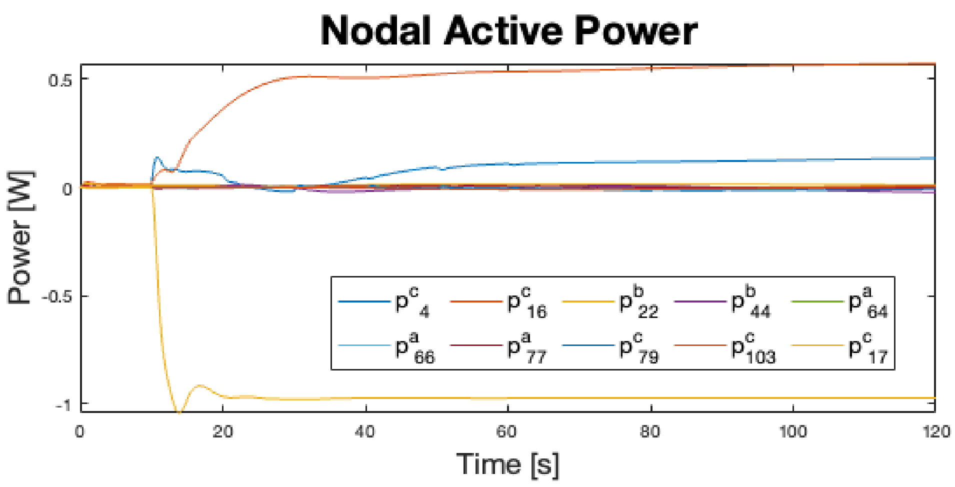


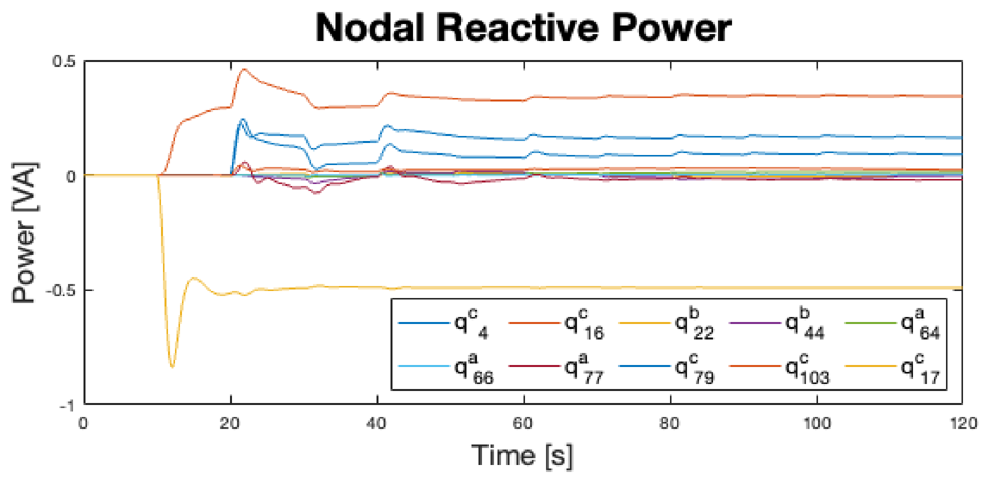
MPC

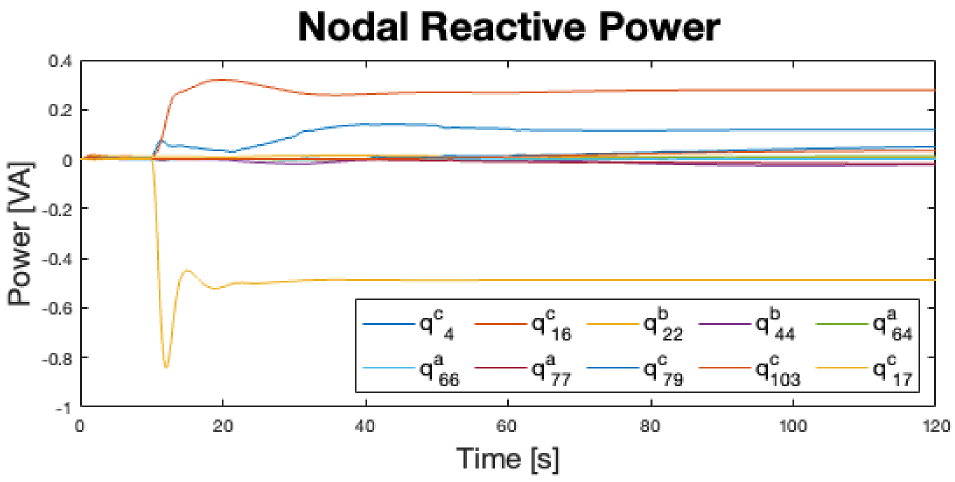


PI control	MPC









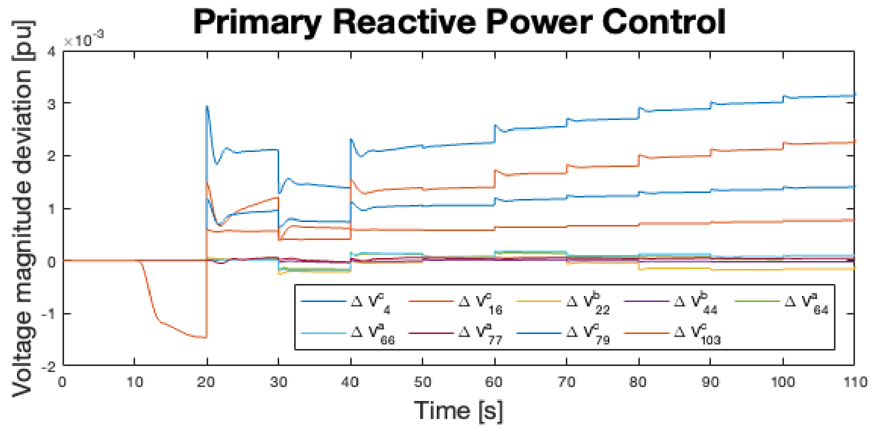
MPC also reduces deviations and oscillations in power injections

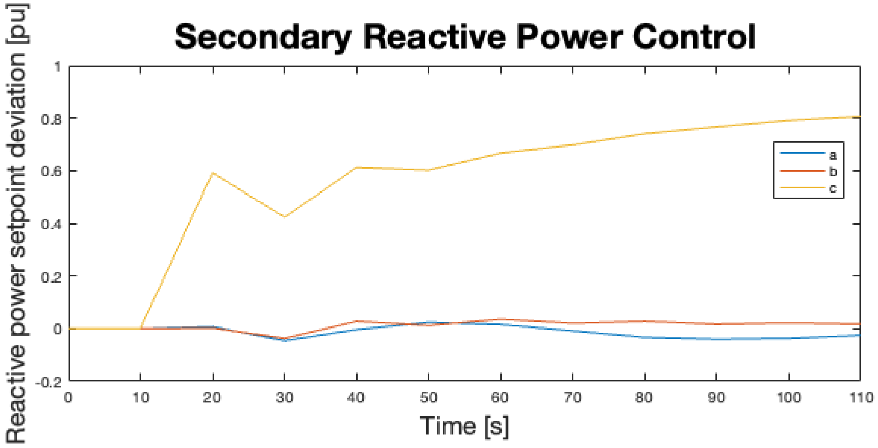
Comparing control inputs

- **PI control actuation**
  → Averages voltage deviations over network
  → Applies same setpoint change to all generators

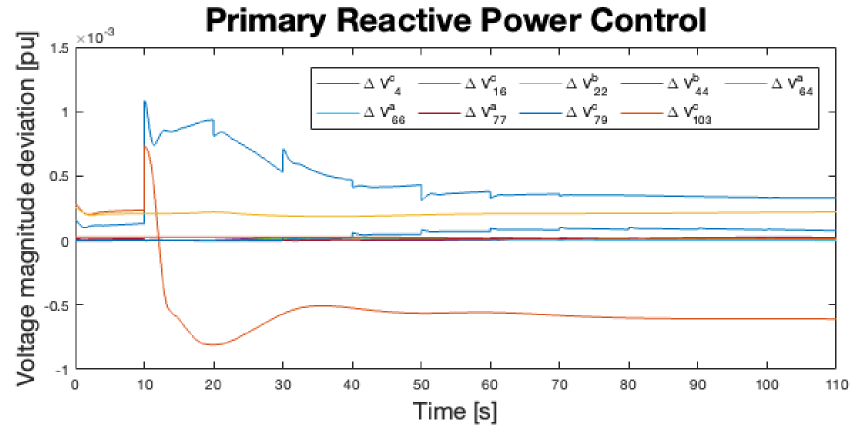
- **MPC actuation**
  → Change setpoints for generators individually
  → Rely more on IBRs closer to load step
  → More efficient dispatch

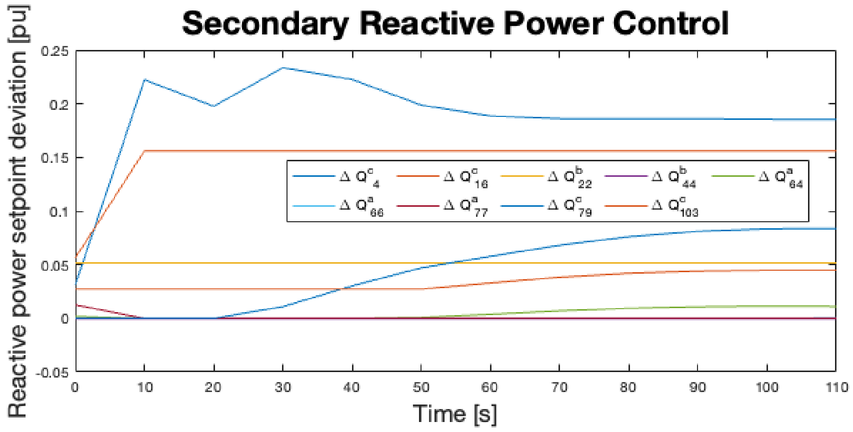
PI control





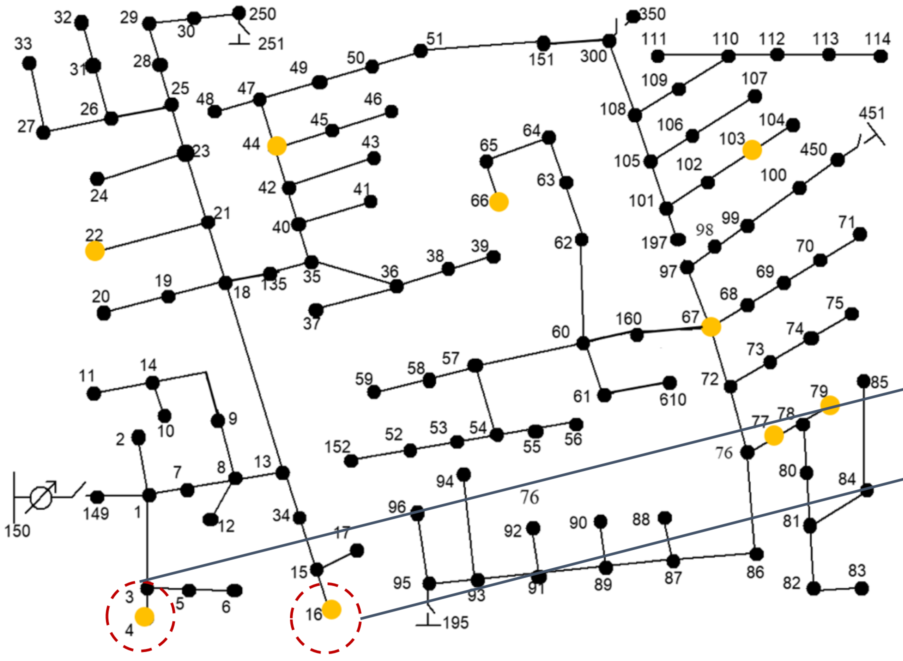
MPC

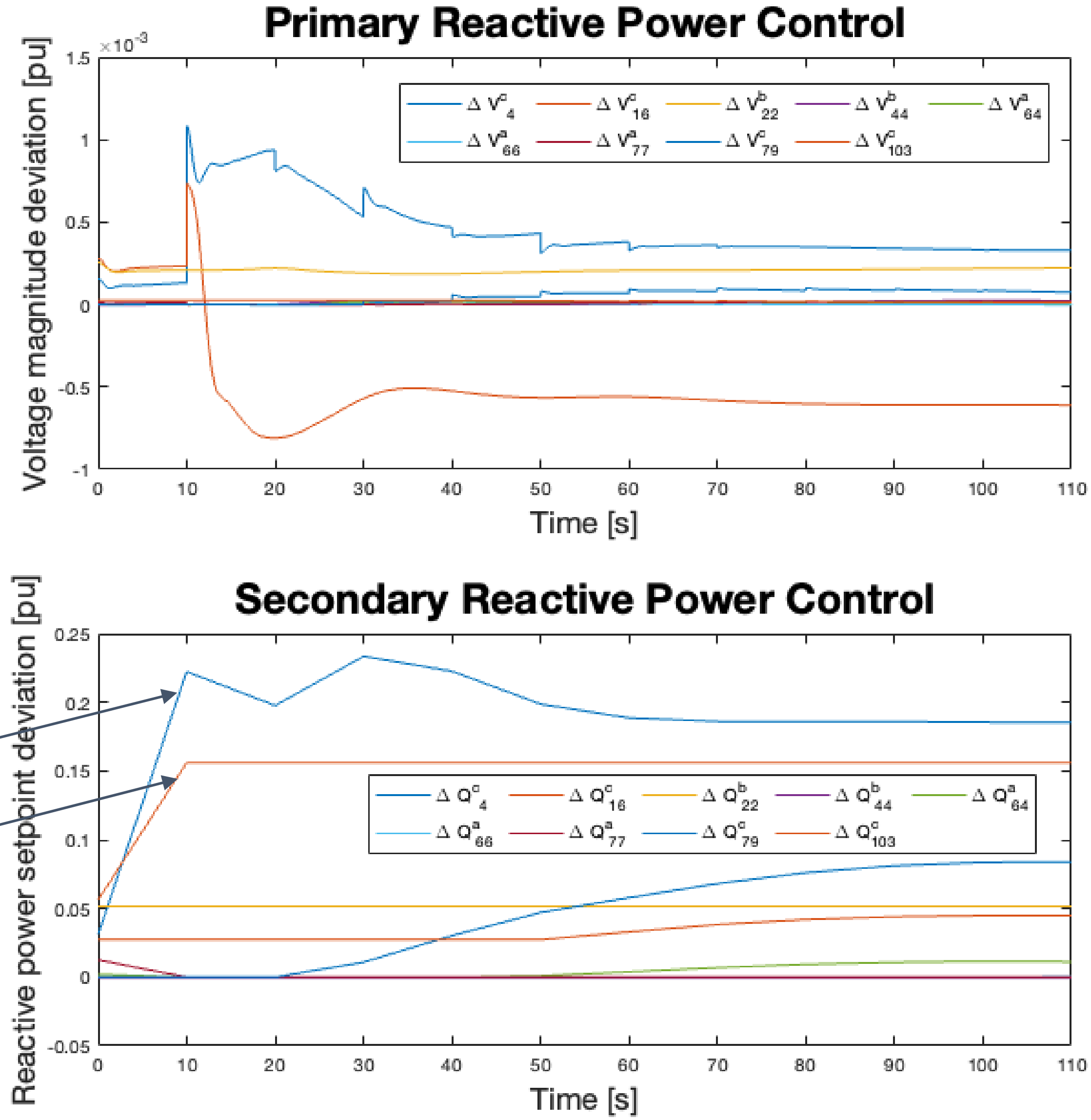




MPC control inputs

- **MPC actuation**
  → Change setpoints for generators individually
  → Rely more on IBRs closer to load step
  → More efficient dispatch





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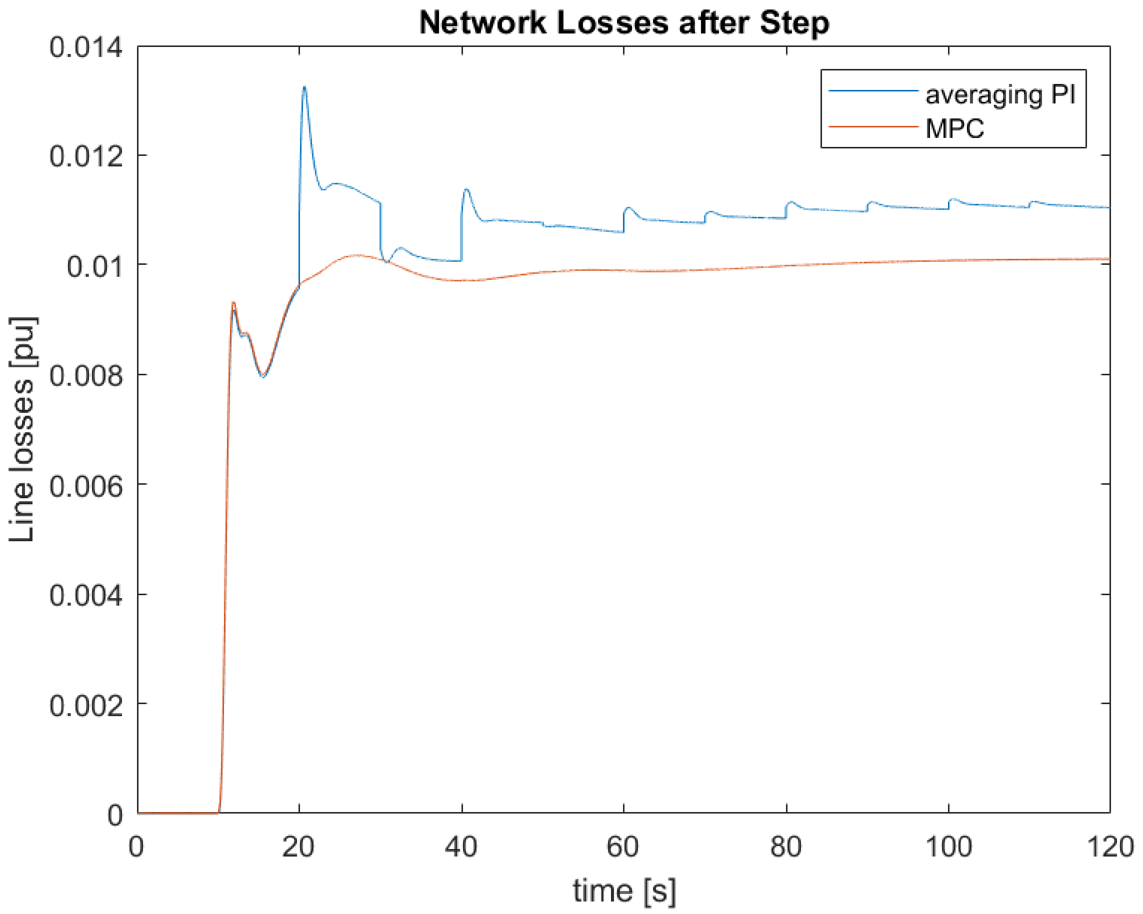
Efficient dispatch reduces power losses by ~11%



- MPC with circuit dynamics is better suited for future grids with IBRs
  → Actively reduces line losses & oscillations
  → Fast, stable response to disturbances

- Distributed MPC algorithm
  → Modular, localized, scalable computation

- Future work
  → Test on larger grids with real data & more IBRs
  → Consider other load models
  → Extend to cases with limited visibility/measurements