Estimation of Cumulative Prospect Theory-based passenger behavioral models for dynamic pricing & transactive control of shared mobility on demand

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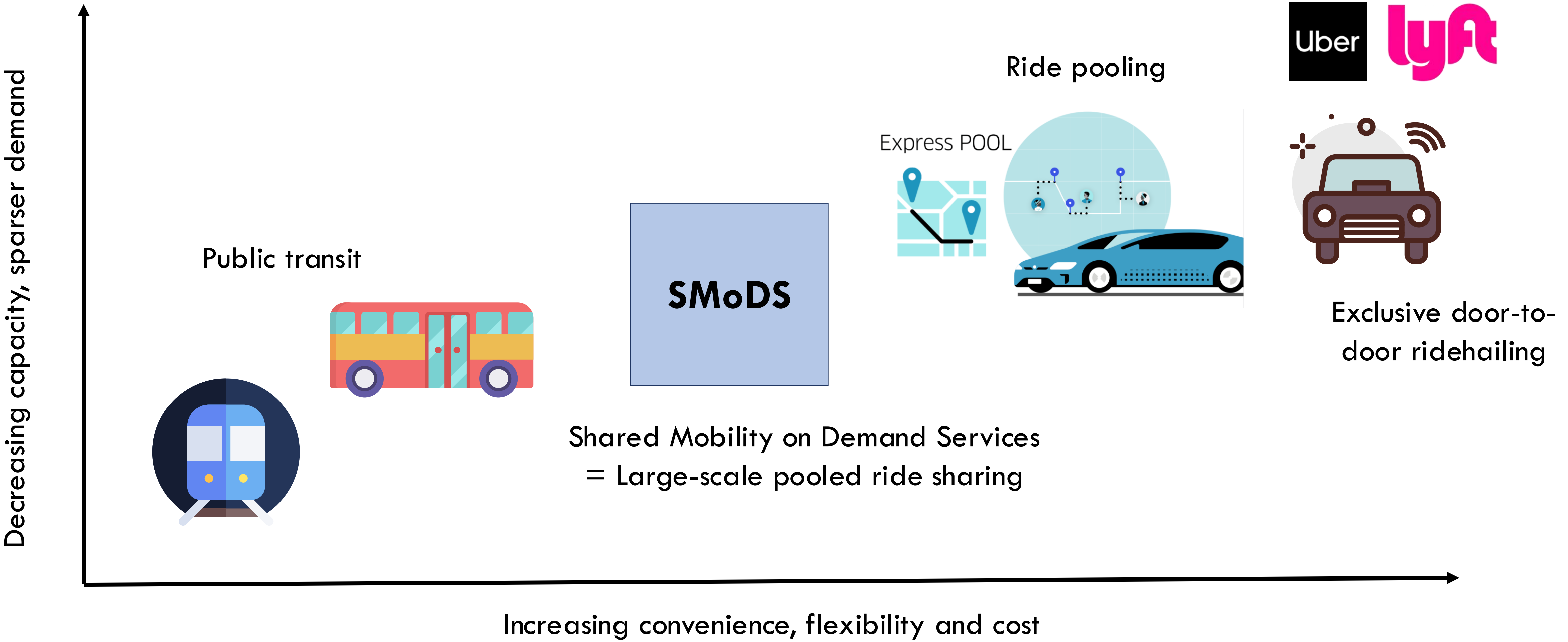
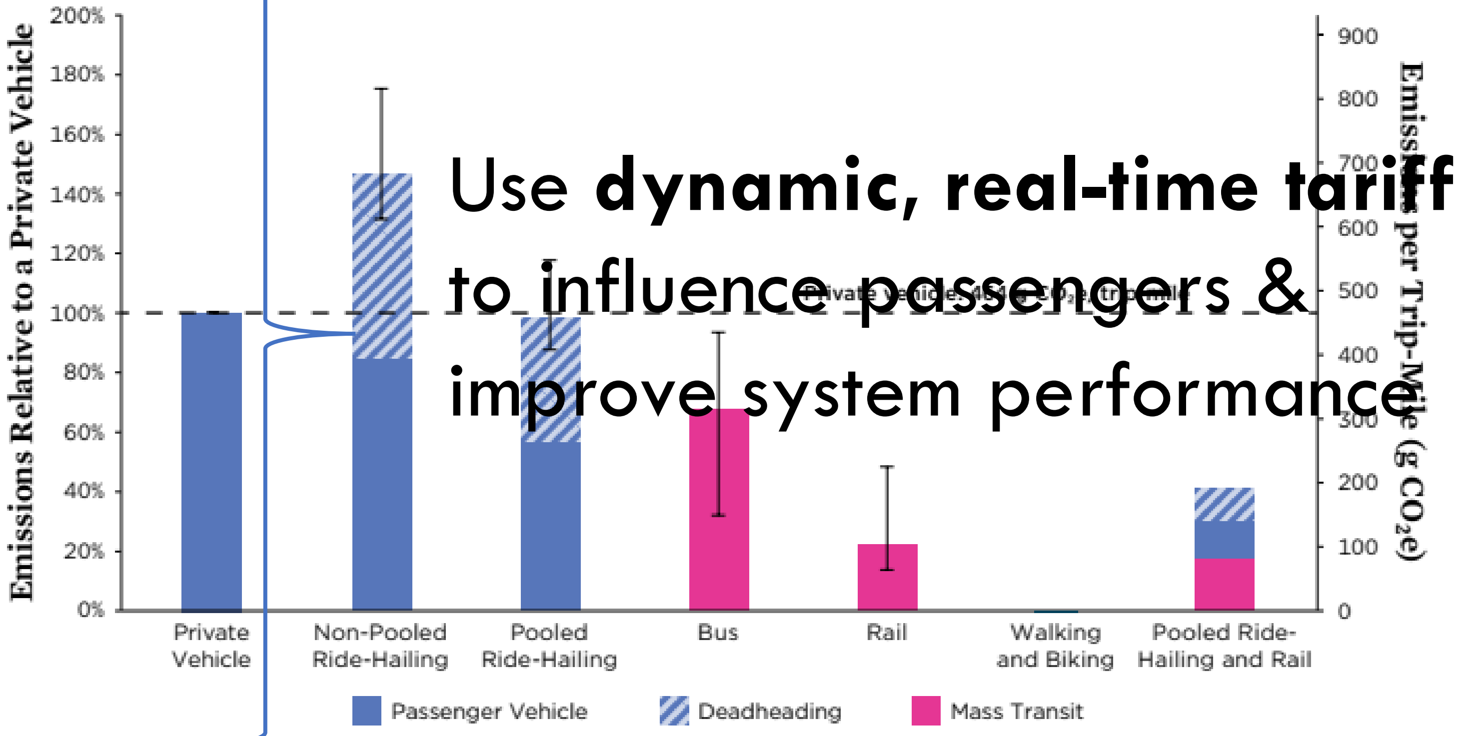
Increasing proliferation of shared mobility services



Impacts of ridesharing and ride pooling

- Affordable, customizable & convenient
- Fleet utilization, efficiency
- Lower overall travel times
- Traffic congestion
- GHG emissions

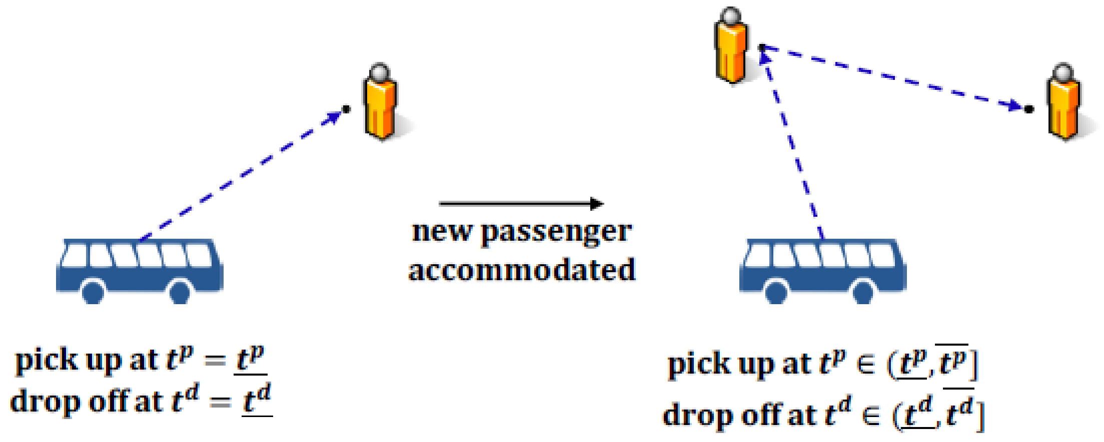


Use **dynamic, real-time tariffs** to influence passengers & improve system performance

[1] Anair, Don et al. 2020. Ride-Hailing's Climate Risks: Steering a Growing Industry toward a Clean Transportation Future. Cambridge, MA: Union of Concerned Scientists.

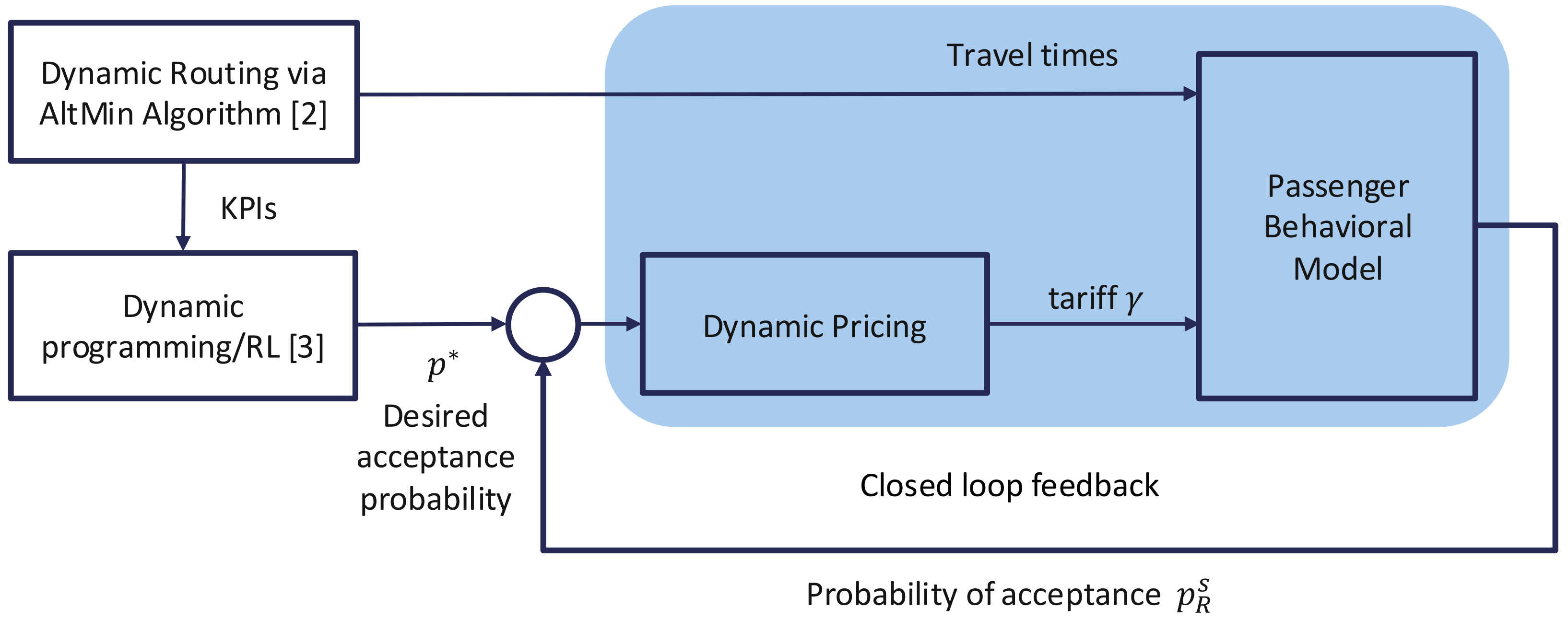
Decision making under increased uncertainty

- SMoDS → Higher travel time uncertainty
- Conventional Expected Utility Theory insufficient



***Goal**: Develop behavioral model to accurately describe passengers' risk preferences towards travel time uncertainty with SMoDS*

Our proposed complete, integrated solution for SMoDS

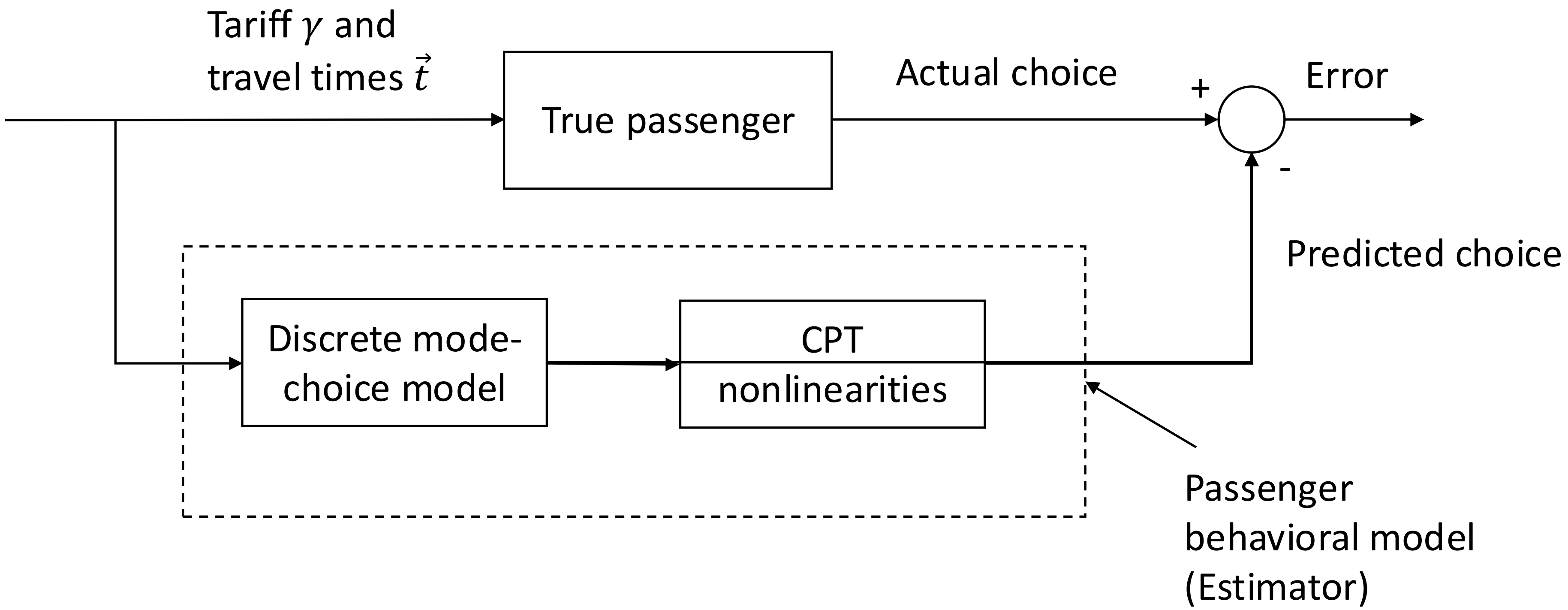


Model identification | **Model applications**

Survey design → Model estimation → Sensitivity & robustness analysis → Closed loop transactive control

[2] Y. Guan, A. M. Annaswamy, and H. E. Tseng, “A Dynamic Routing Framework for Shared Mobility Services,” ACM Transactions on Cyber-Physical Systems, 2020.
[3] Y. Guan, A. M. Annaswamy, and H. Eric Tseng, “Towards Dynamic Pricing for Shared Mobility on Demand using Markov Decision Processes and Dynamic Programming,” IEEE 23rd International Conference on Intelligent Transportation Systems (ITSC), 2020.
[4] **Nair, V. J.**, Guan, Y., Annaswamy, A. M., Tseng, H. E., & Singh, B., “Sensitivity Analysis of Passenger Behavioral Model for Dynamic Pricing of Shared Mobility on Demand”, *Transportation Research Part A: Policy & Practice*, 2021 *(under review)*.
[5] **Nair, V. J.**, Annaswamy, A. M., Tseng, H. E., & Singh, B., “Estimation of CPT behavioral model for dynamic pricing & transactive control of shared mobility on demand”, *Transportation Research Part B: Methodological*, 2021 *(in preparation)*.

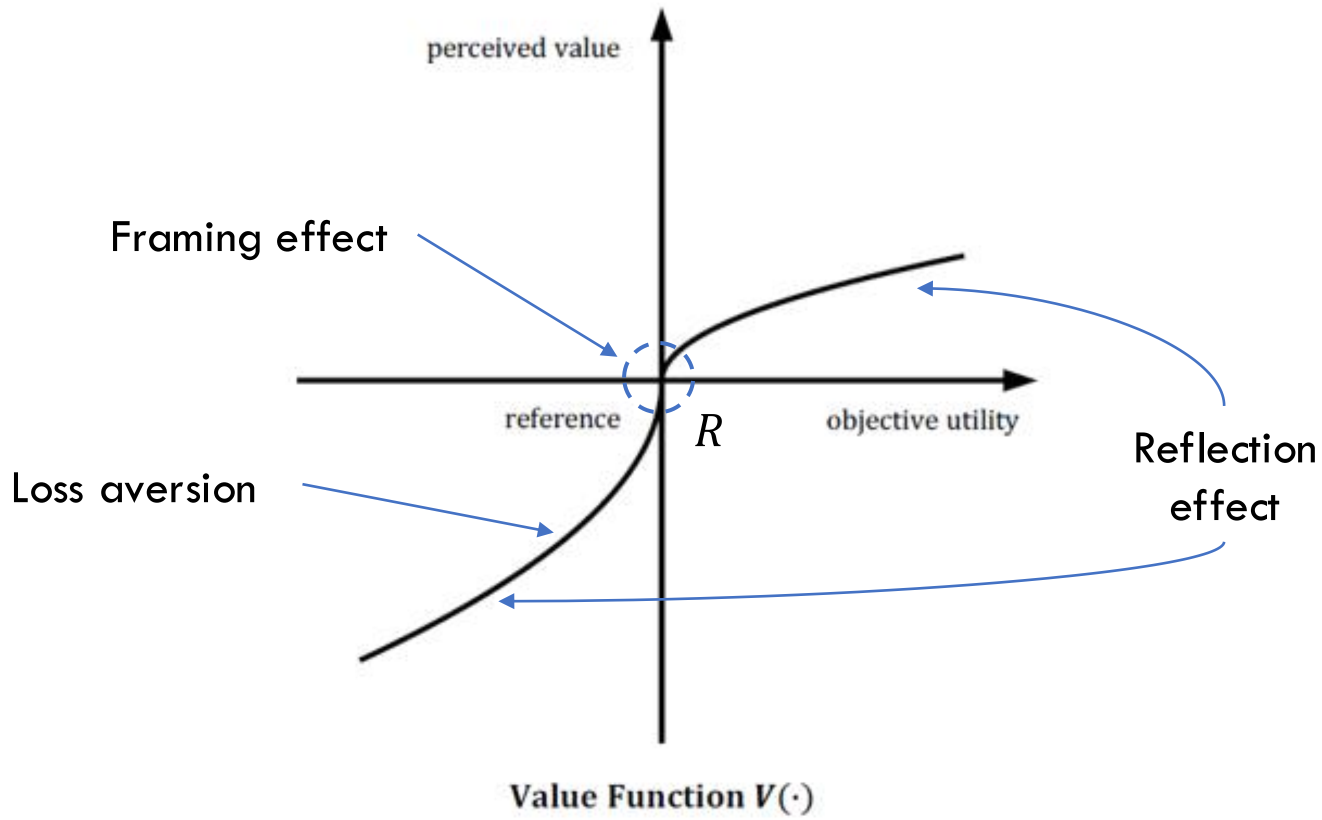
Passenger behavioral model overview



Challenging to model irrational human behavior

New tool proposed: Prospect Theory

Cumulative Prospect Theory (CPT) model



Value function

$$V(u) = \begin{cases} (u-R)^{\beta^+}, & u \geq R \\ -\lambda(R-u)^{\beta^-}, & otherwise \end{cases}$$

u: Objective utility; V: Perceived value

$$0 < \beta^+, \beta^- < 1, \lambda > 0$$

[7] A. Tversky and D. Kahneman, "Advances in prospect theory: Cumulative representation of uncertainty," J Risk Uncertainty, vol. 5, no. 4, pp. 297–323, Oct. 1992.
[8] Guan, Y., Annaswamy, A. M., & Tseng, H. E. Tseng. Cumulative prospect theory based dynamic pricing for shared mobility on demand services. In 2019 IEEE 58th Conference on Decision and Control (CDC) (pp. 2239-2244). IEEE.

Cumulative Prospect Theory (CPT) model

Probability distortion

$$\pi(p) = \exp(-[-\ln(p)]^{\alpha}),\ 0 \leq \alpha \leq 1$$

SMoDS with n possible arrival times

$$u_1 < u_2 \ldots < u_n$$

Subjective weights:

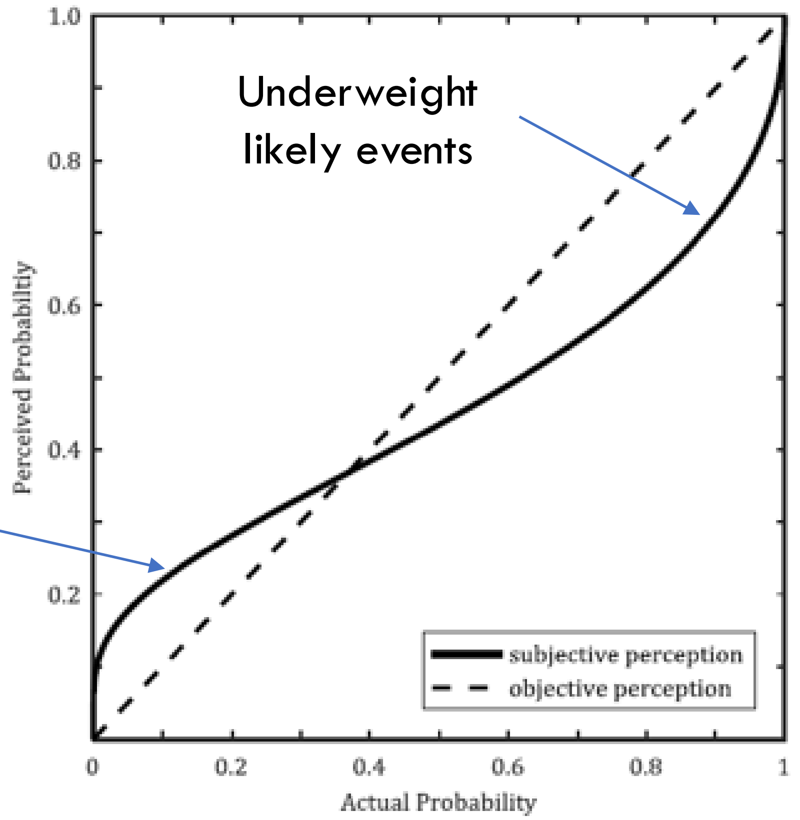
$$w_i = \begin{cases} \pi[F_U(u_i)] - \pi[F_U(u_{i-1})], u_i < R \\ \pi[1 - F_U(u_{i-1})] - \pi[1 - F_U(u_i)], u_i \geq R \end{cases}$$

Subjective utility: $U_R^S = \sum_{i=1}^{n} w_i V(u_i)$

Passenger's subjective probability of acceptance:

$$p_R^S = \frac{e^{U_{R,SMoDS}^S}}{\sum_j e^{U_{R,j}^S}}$$

Overweight rare events



Probability Weighting Function $\pi(\cdot)$

[9] M. O. Rieger, M. Wang, and T. Hens, "Risk Preferences Around the World," Management Science, vol. 61, no. 3, pp. 637–648, Feb. 2014.
[10] C. Bernard and M. Ghossoub, "Static portfolio choice under Cumulative Prospect Theory," Math Finan Econ, vol. 2, no. 4, pp. 277–306, Mar. 2010.
[11] T. Tanaka, C. F. Camerer, and Q. Nguyen, "Risk and Time Preferences: Linking Experimental and Household Survey Data from Vietnam," American Economic Review, vol. 100, no. 1, pp. 557–571, Mar. 2010.
[12] R.-C. Jou and K.-H. Chen, "An application of cumulative prospect theory to freeway drivers' route choice behaviours," Transportation Research Part A: Policy and Practice, vol. 49, pp. 123–131, Mar. 2013.
[13] S. Wang and J. Zhao, "Risk preference and adoption of autonomous vehicles," Transportation Research Part A: Policy and Practice, vol. 126, pp. 215–229, Aug. 2019.

Model determination using CPT – Step 1: Survey Design

Current travel behavior

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Mode-choice scenarios

↓

Customized choice games for travel risk attitudes

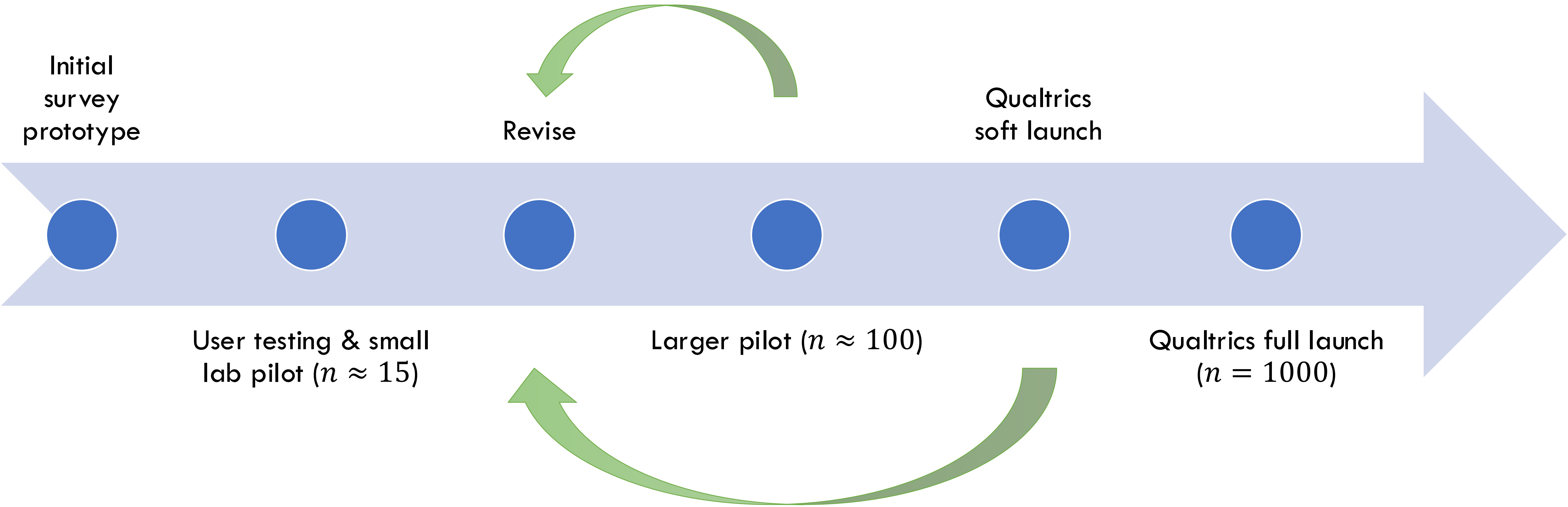
↓

Standardized lottery questions for financial risk

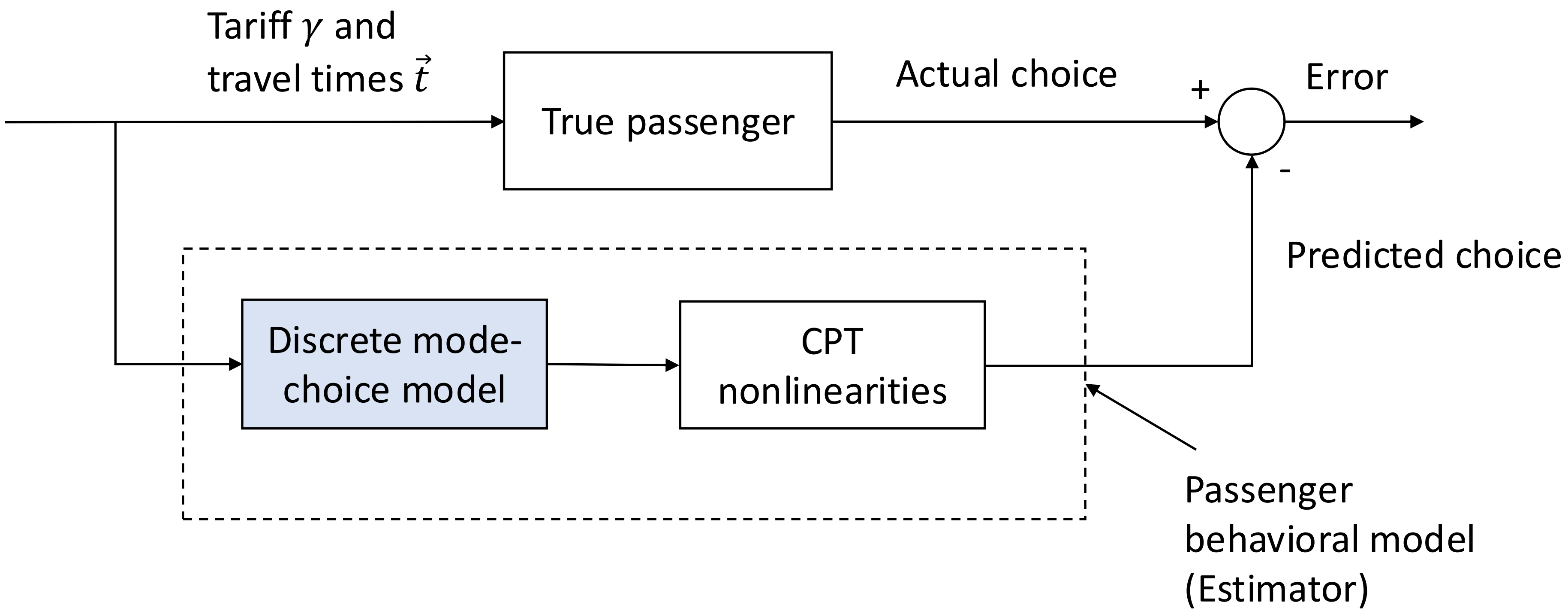
↓

Demographic questions

Survey design process



Model determination using CPT – Step 2: Mode-choice Modeling



Mode choice model

- Mode-specific *objective* utility for each alternative i:

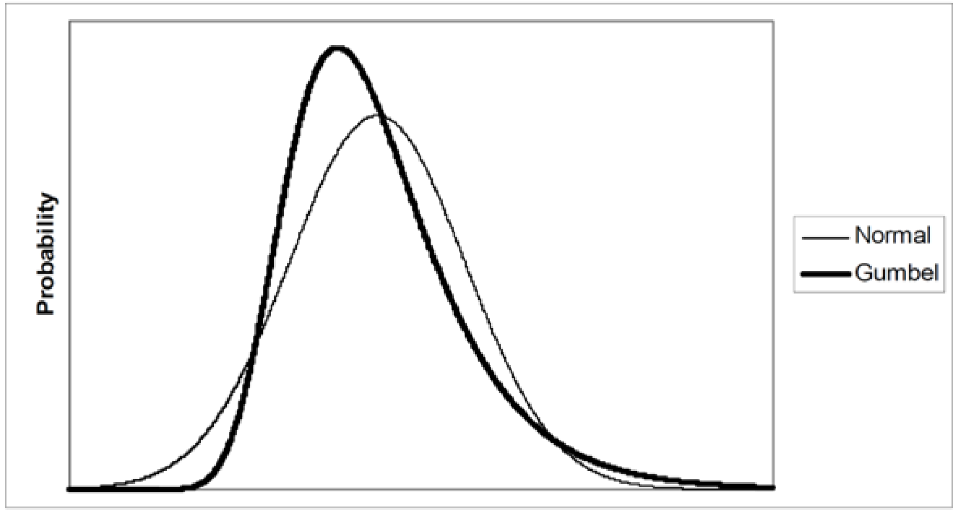
$$u_i = a_{walk} t_{walk,i} + a_{wait} t_{wait,i} + a_{ride,i} t_{ride,i} + b\gamma_i + ASC_i + \epsilon_i$$

$i \in$ {Public transit, Exclusive ridehailing, Pooled ridesharing}

Assume same across all modes

In-vehicle travel time Trip tariff Alternative specific constant Random error term

- Set $ASC_{transit} = 0$
- Multinomial logit model:
  - Computationally easier than other models (e.g. probit)



[4] K. Train, Discrete Choice Methods with Simulation. Jun. 2009.
[5] M. E. Ben-Akiva and S. R. Lerman, Discrete choice analysis: theory and application to travel demand, ser. MIT Press series in transportation studies. Cambridge, Mass: MIT Press, 1985, no. 9.

Mode choice scenarios

- Stated preferences
- Factorial design of experiments
- Main effects screening design
  - Negligible interaction terms & higher order effects

Travel option	**Total trip cost ($)**	**Walking time (min)**	**Waiting time (min)**	**In-vehicle riding time (min)**
Public transit: Subway (T)	2.4	14	5	8
Exclusive rideshare	6.4	0	4	12
Pooled rideshare	3.23	4	3	15

- ○ Public transit: Subway (T)
- ○ Exclusive rideshare
- ○ Pooled rideshare

[6] J. J. Louviere, D. A. Hensher, and J. D. Swait, "Stated Choice Methods: Analysis and Applications," Sep. 2009.

Mode choice logit models

- Standard logit

$$P_i = \frac{e^{u_i}}{\sum_{j=1}^{3} e^{u_j}}$$

Mode i, j

Passenger n

- Mixed logit
  - Conditional logit probability $L_{ni}(\beta) = \frac{e^{u_{ni}(\beta)}}{\sum_j e^{u_{nj}(\beta)}}$
  - $P_{ni} = \int L_{ni}(\beta) f(\beta) d\beta$
  - Common mixing distributions: $\beta \sim N(\mu, \sigma^2)$, $l\,n(\beta) \sim N(\mu, \sigma^2)$ etc.

[14] K. Train, "Mixed logit with a flexible mixing distribution," Journal of choice modelling, vol. 19, pp. 40–53, Jun. 2016.
[15] K. E. Train, "Recreation Demand Models with Taste Differences over People," Land Economics, vol. 74, no. 2, p. 230, May 1998.
[16] D. Revelt and K. Train, "Mixed Logit with Repeated Choices: Households' Choices of Appliance Efficiency Level," Review of Economics and Statistics, vol. 80, no. 4, pp. 647–657, Nov. 1998.
[17] F. S. Koppelman and C. Bhat, "A self instructing course in mode choice modeling: multinomial and nested logit models," 2006.

Mode choice estimation

- Choice probabilities → **likelihood function**

$$l(\beta) = \prod_{\forall n} \prod_{\forall j} (P_{nj}(\beta))^{\delta_{nj}}$$

δ_{nj} = 1 if mode j is chosen by, passenger n, 0 otherwise

~~Exact MLE~~

$$\cancel{P_{ni} = \int L_{ni}(\beta) f(\beta) d\beta}$$

- Maximum simulated likelihood estimator (MSL):

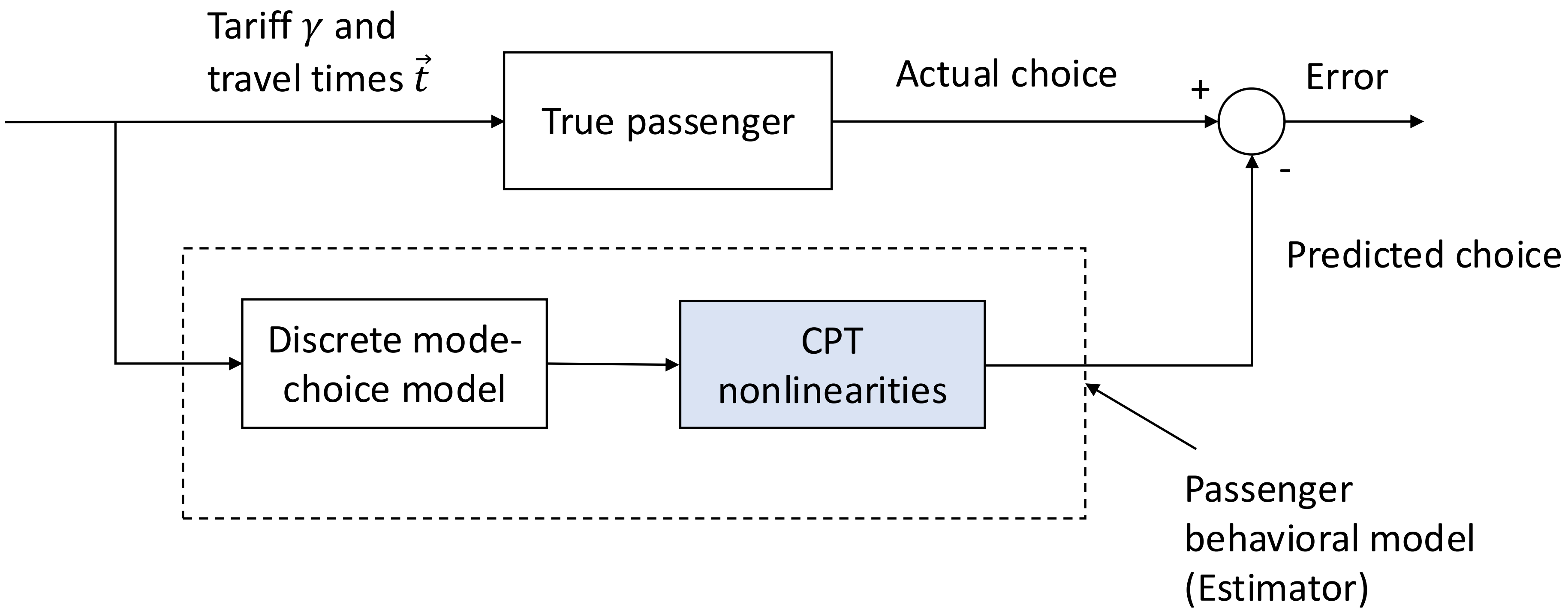
$$\tilde{l}(y_i|\theta) = \frac{1}{R} \sum_{r=1}^{R} L_{ni}(\beta_r)$$

$$\tilde{\theta} = argmax_{\theta} \frac{1}{N} \sum_{i=1}^{N} ln\, \tilde{l}(y_i|\theta)$$

- Consistent
- Asymptotically normal
- Efficient
- Equivalent to exact MLE

$\tilde{l}$: Unbiased simulated likelihood function
y_i: Sample observations
$\beta_r \sim f(\beta|\theta)$: Simulations (randomly drawn parameters)
θ: Mixing distribution parameters (e.g. μ, σ)
R: No. of draws per observation
N: No. of observations (choice scenarios)

Model determination using CPT – Step 3: CPT Nonlinearities



CPT choice scenarios for risk attitudes

Trafield (SMoDS)

Pure gain

10% chance	Win $10
90% chance	Win $100

What is the maximum amount (in $) you would pay to play in this lottery?

Pure loss

u_1 → 60% chance	Loss of $80
u_2 → 40% chance	No loss or win

u_{survey} → What is the maximum amount (in $) you would pay to avoid this lottery?

Mixed outcomes

50% chance	Loss of $25
50% chance	Win $X

What is the minimum amount X (in $) that would make this lottery acceptable to you?

CPT model estimation

Certainty equivalence (CE) method

$$CE_{pred} = w_1 V(u_1) + w_2 V(u_2) = \hat{U}^{s}_{R,SMoDS}$$

$$CE_{true} = U^{s}_{R,\,A} = U^{s}_{R,\,SMoDS} = V\left(u_{survey}\right)$$

Error for each choice scenario

$$CE_{true} - CE_{pred} = U^{s}_{R,\,SMoDS} - \hat{U}^{s}_{R,\,SMoDS} = e$$

Constrained nonlinear least squares *for each respondent*

$$\min \sum e^2$$
$$s.t.\, 0 \le \alpha, \beta^+, \beta^- \le 1, \lambda \ge 1$$

Normalize by CE_{true} or $\max\{|u_1|, |u_2|\}$

Rescale λ

Modified weighting function

$$\pi(p) = \frac{p^\alpha}{(p^\alpha + (1-p)^\alpha)^{\frac{1}{\alpha}}}$$

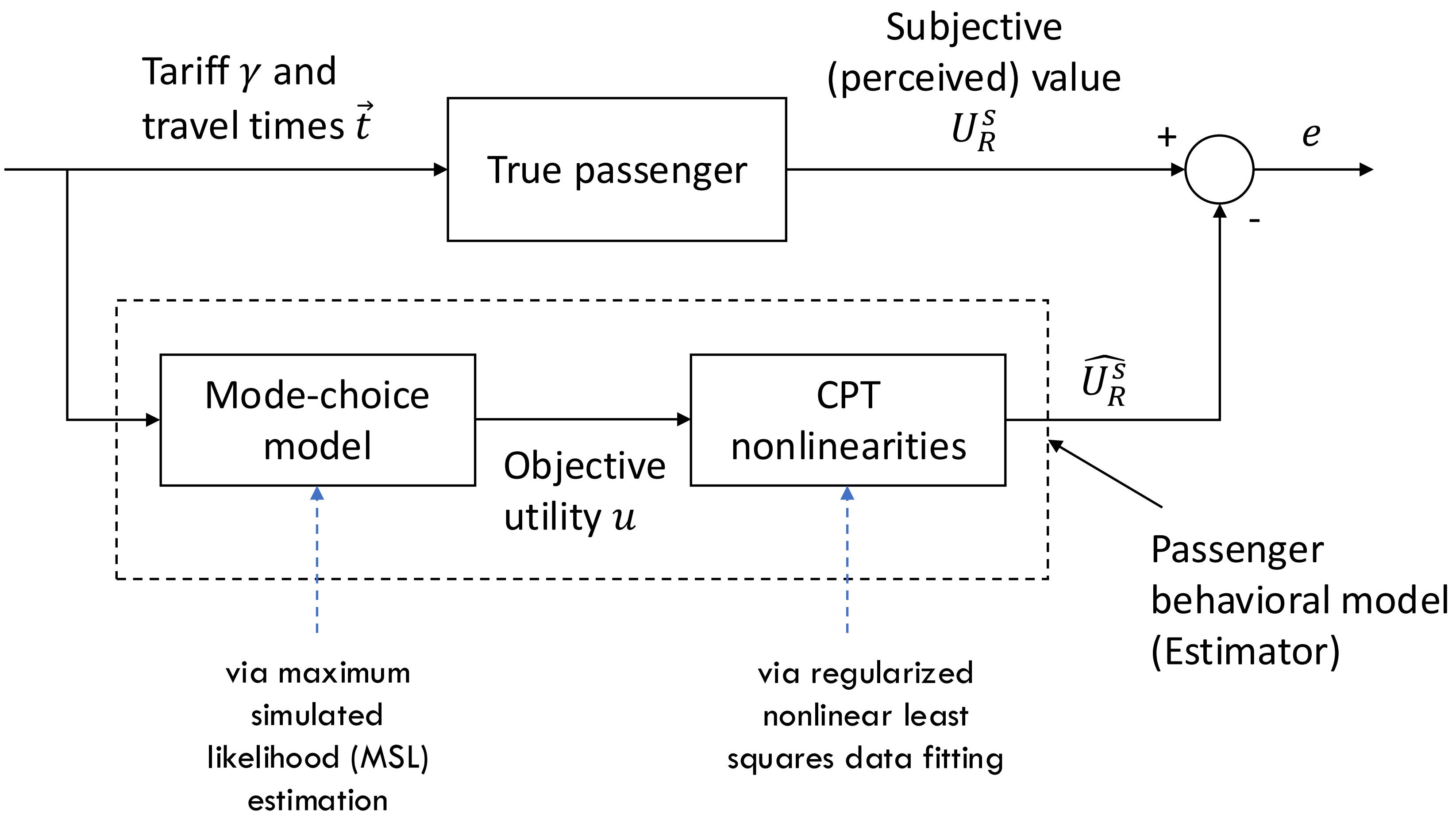
Numerical solver settings

$\alpha \to \alpha^+, \alpha^-$

L2 norm regularization

$$\min \sum e_n^2 + \lambda_1 \left|\left|\Theta - \overline{\Theta}\right|\right|^2 + \lambda_2 \left|\left|\Theta - \underline{\Theta}\right|\right|^2 + \lambda_3 \left|\left|\Theta - \Theta_{lottery}\right|\right|^2$$
$$s.t.\, 0 \le \alpha^+, \alpha^-, \beta^+, \beta^-, \lambda \le 1$$
$$\Theta = [\alpha^+, \alpha^-, \beta^+, \beta^-, \lambda]$$

Estimation process overview



Model determination using CPT – Step 4: Validation

Fixed coefficients

Random coefficients

Parameter	Mean μ	SE	SD σ	SE
a_{walk}	-0.0586	0.0053	0.1412	0.0079
a_{wait}	-0.0113	0.0182	0.1491	0.0356
$a_{ride,\ transit}$	-0.0105	0.0013	0.0284	0.0017
$a_{ride,\ exclusive}$	-0.0086	0.0014	0.0058	0.0010
$a_{ride,\ pooled}$	-0.0186	0.0013	0.0095	0.0007
b	-0.0518	0.0050	0.0597	0.0042
$ASC_{exclusive}$	-2.5926	0.1800	2.3034	0.1558
ASC_{pooled}	-2.2230	0.1497	1.8175	0.1530
Log-likelihood value at convergence	-6.5350×10^3			
Likelihood ratio index ρ	0.4338			

Likelihood ratio index

$$\rho = 1 - \frac{LL(\widehat{\beta})}{LL(0)}$$

$$\boxed{* \ ASC_{transit} = 0}$$

Value of time (VOT)

$$VOT_{mode} = \frac{\frac{\partial U}{\partial t}}{\frac{\partial U}{\partial \gamma}} = \frac{a_{mode}}{b} \times 60 \ \ [\$/h]$$

Trip leg or mode	VOT (in \$/h)
Walking	67.8702
Waiting	13.1480
Transit ride	12.1703
Exclusive ride hailing	9.9466
Pooled ride sharing	21.5549

https://www.bls.gov/regions/new-england/news-release/occupationalemploymentandwages_boston.htm

Surveyed passengers do show CPT behavior

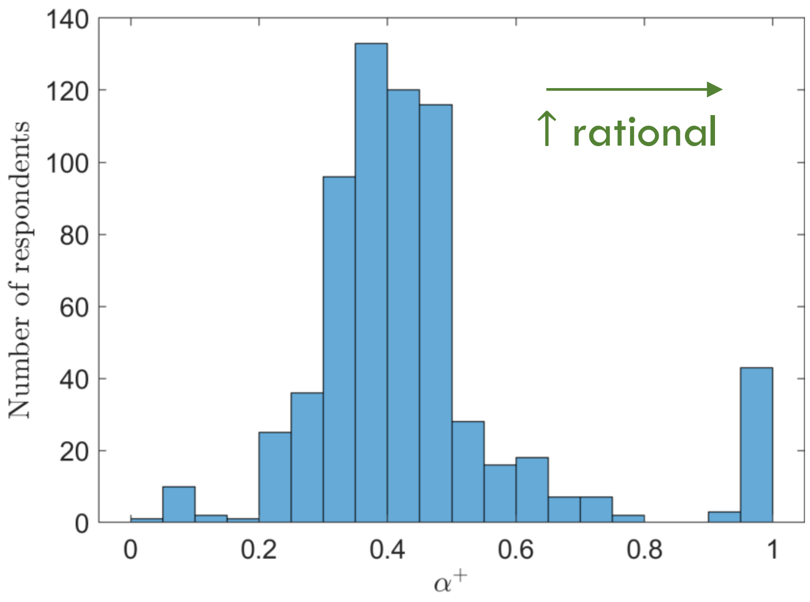
- Screen out potentially erroneous respondents
  - Internal validity
  - Monotonicity of probabilities & outcomes

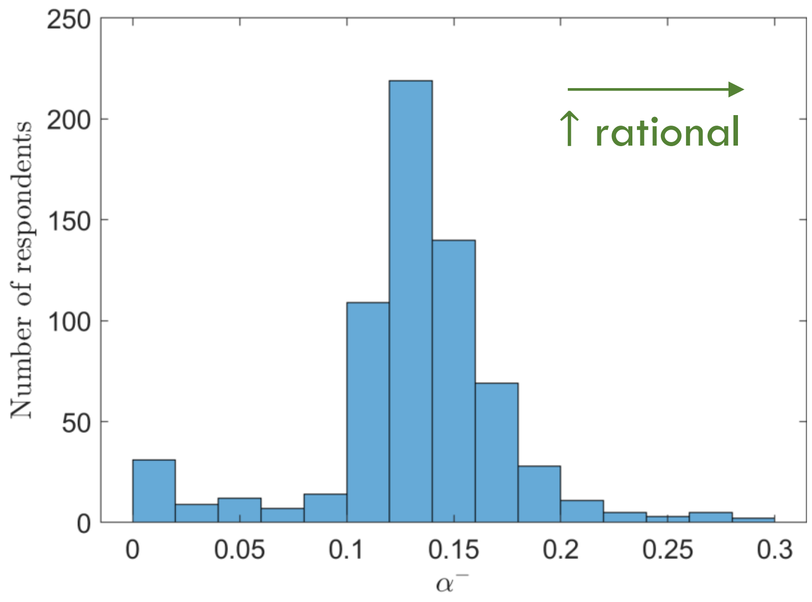
→ 664 valid respondents

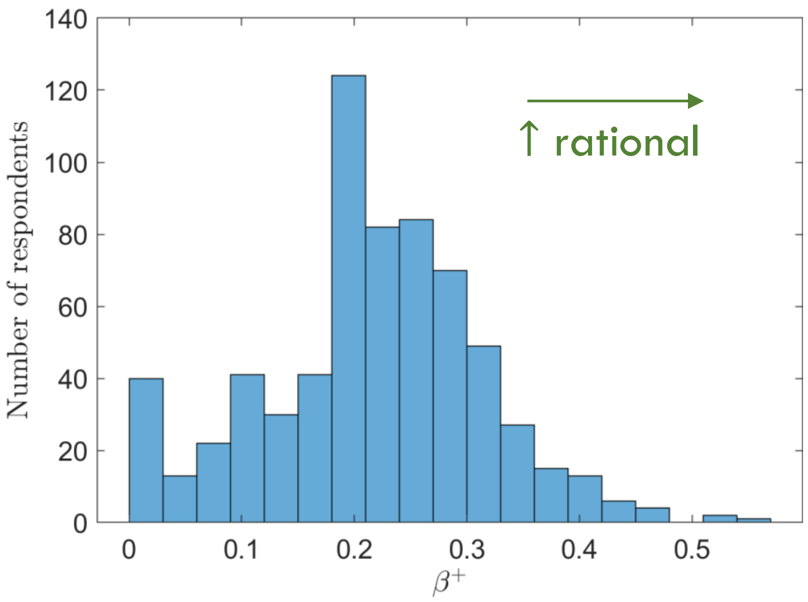
- Detect CPT-like behaviors using pairs of lottery questions

CPT effect tested	% of valid responses
Reflection effect	95.03
Probability overweighting between	
10% and 60% probability	62.56 %
60% and 90% probability	40.51 %
10% and 90% probability	51.05 %
Any probability weighting	72.44 %
Mean gain/loss ratio for mixed outcome lotteries	3.7254
Median gain/loss ratio for mixed outcome lotteries	1.0250

Financial risk CPT parameters

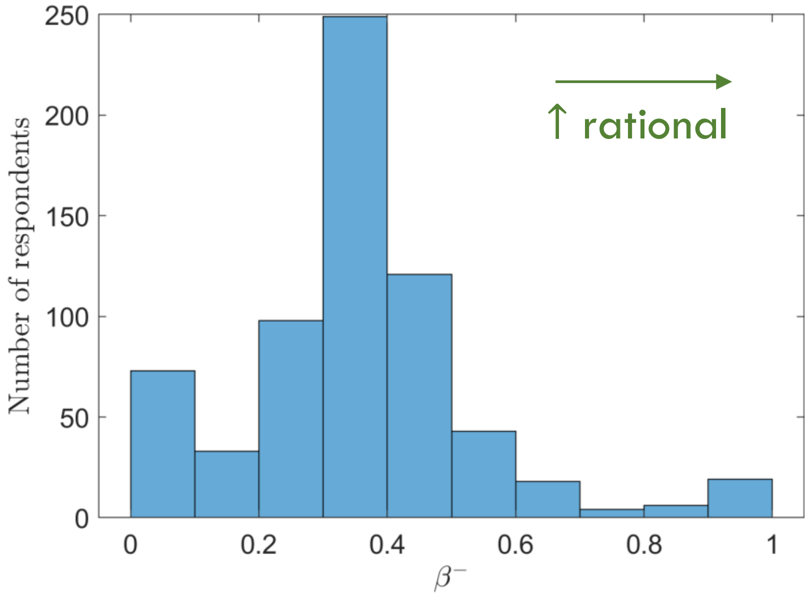


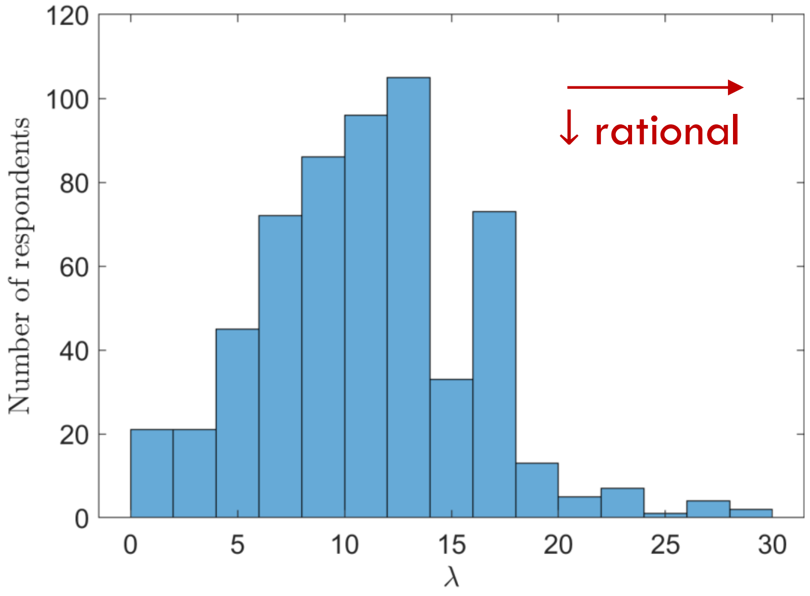




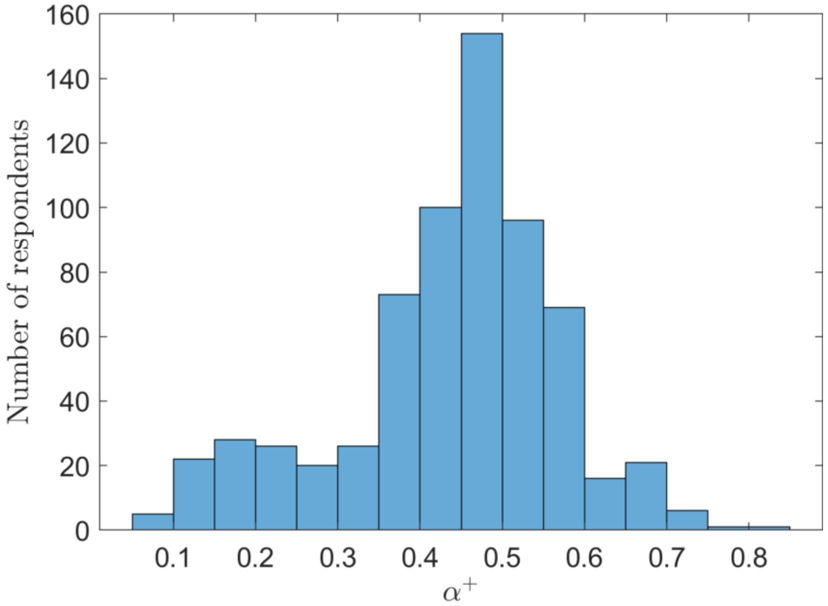
$$V(u) = \begin{cases} (u-R)^{\beta^+}, u \geq R \\ -\lambda(R-u)^{\beta^-}, u < R \end{cases}$$

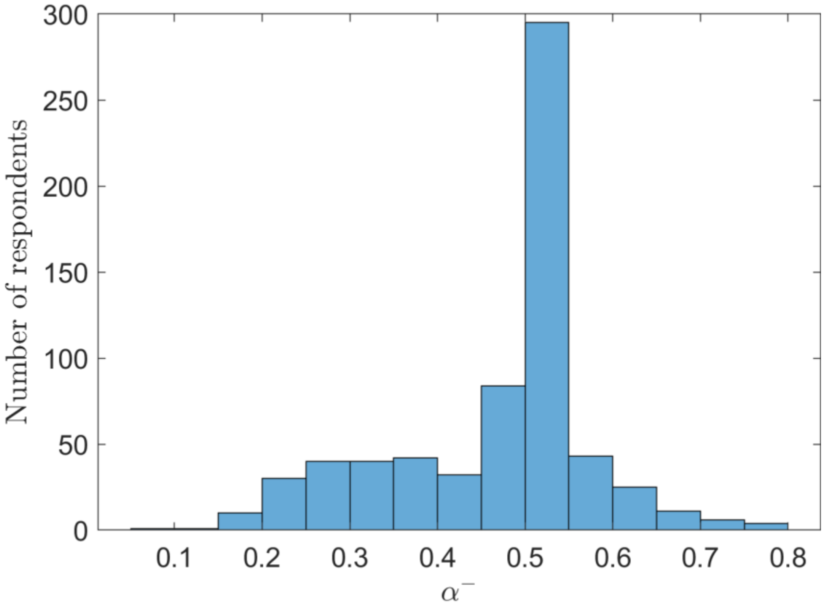
$$\pi_{\pm}(p) = \exp(-[-\ln(p)]^{\alpha_{\pm}})$$

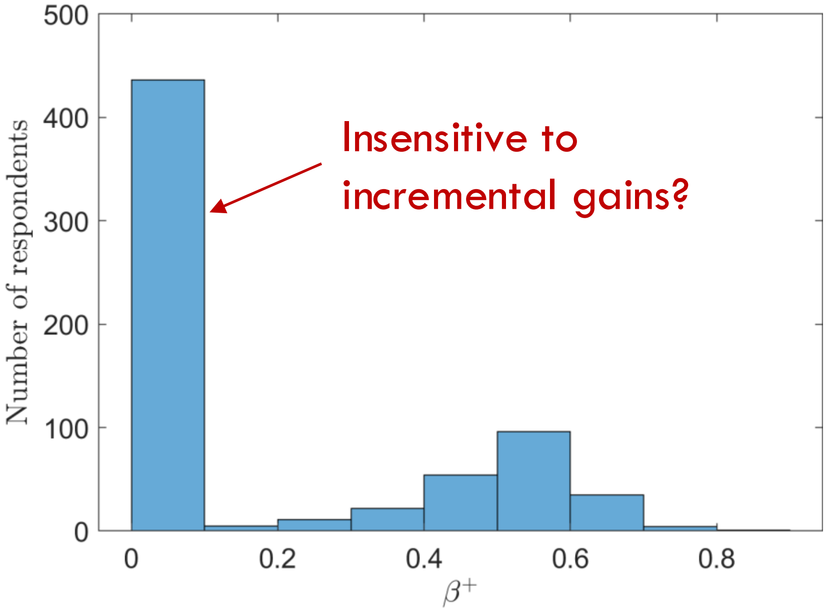




Travel time risk CPT parameters (SMoDS)



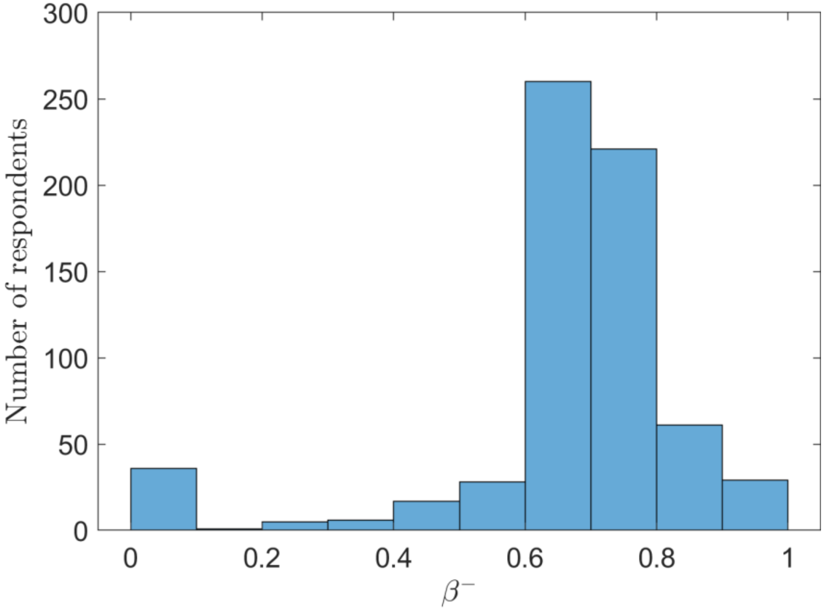


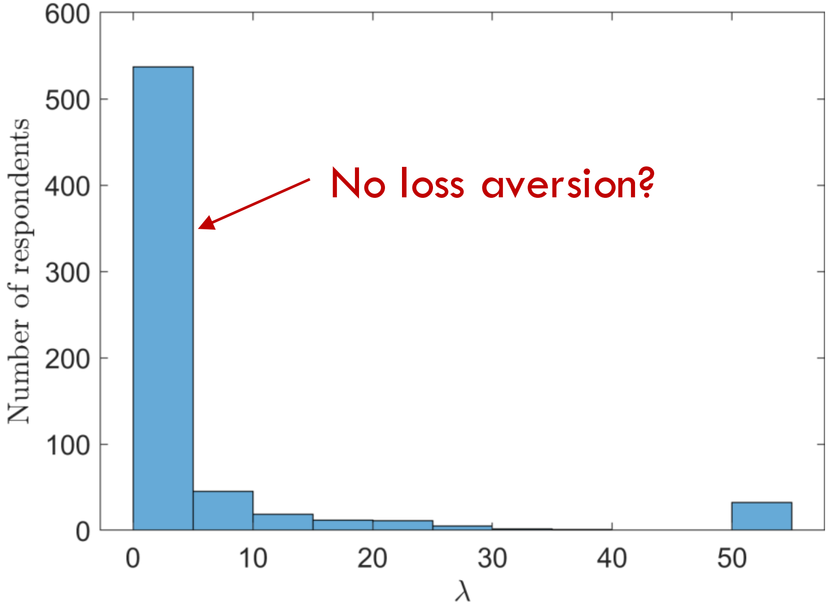


$$V(u) = \begin{cases} (u-R)^{\beta^+}, & u \geq R \\ -\lambda(R-u)^{\beta^-}, & u < R \end{cases}$$

Regularized with $v_1 = v_2 = 1.5$

$$\pi_\pm(p) = \exp(-[-\ln(p)]^{\alpha_\pm})$$

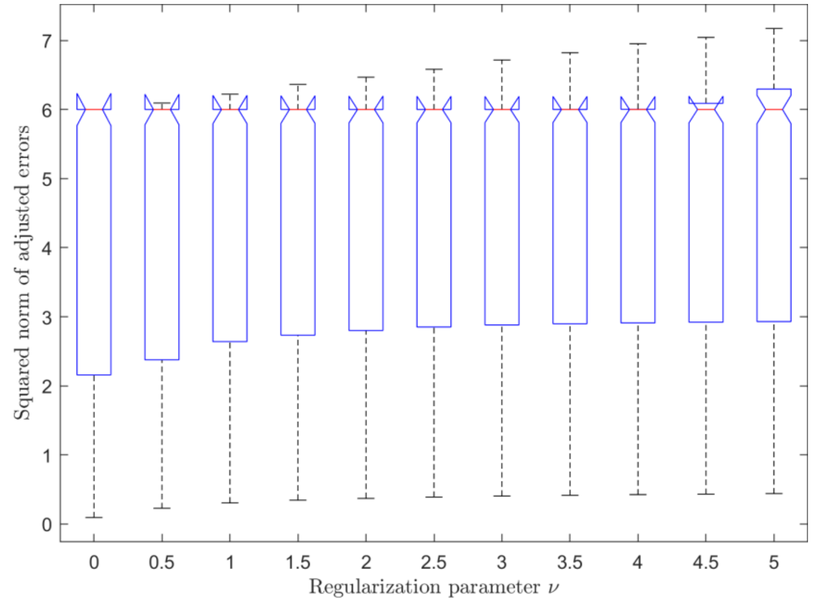




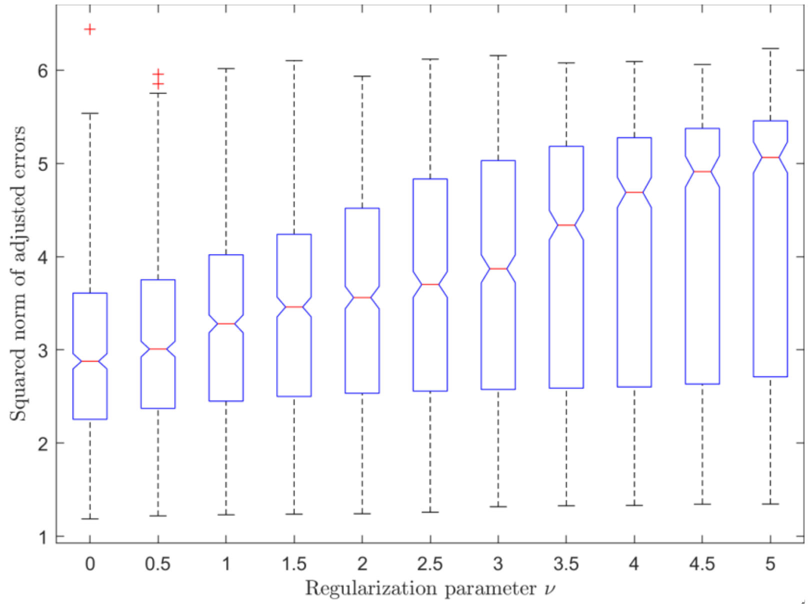
Diagnosing pending issues with CPT estimation

1. Determining reference point R
   - For lotteries, $R = \$0$
   - For travel?
     - Static
     - Dynamic

Dynamic u_0
(Certain alternative)



Dynamic SMoDS
$R = pu_1 + (1 - p)u_2$



Diagnosing pending issues with CPT estimation

1. Determining reference point R

2. Data quality challenges
   - Gains, losses & mixed outcomes
   - Simplifying assumptions
   - Lack of 'ground truth' data

Improve future survey design

Model determination using CPT – Step 5: Sensitivity and robustness

Dynamic pricing: Constrained nonlinear optimization → Real time tariff γ^* → Optimal objective $f(\gamma^*)$

Expected revenue

$$\begin{aligned} \min_{\gamma} \quad & -f(\gamma;\vec{\theta}) \triangleq -\gamma \cdot p_s^R(\gamma;\vec{\theta}) \\ \text{s.t.} \quad g^1: \quad & \underline{\gamma} - \gamma \leq 0 \\ g^2: \quad & \gamma - \overline{\gamma} \leq 0 \end{aligned}$$

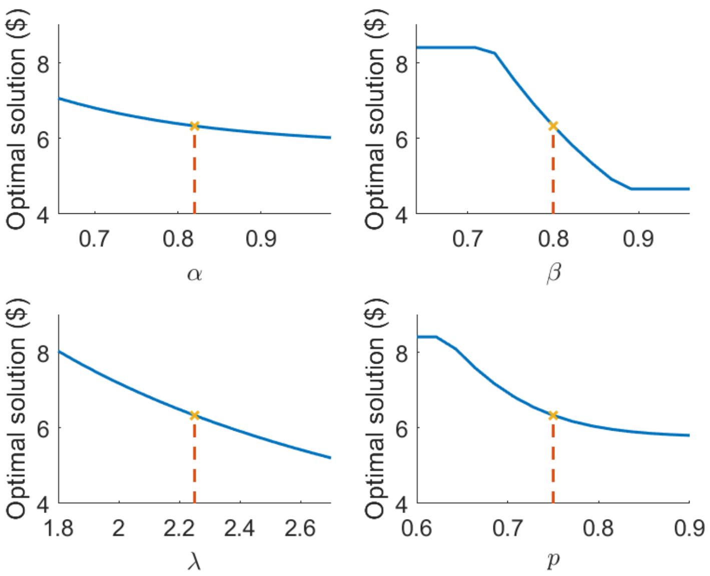
$$\vec{\theta} = [\alpha^+, \alpha^-, \beta^+, \beta^-, \lambda, p]$$

How sensitive are the optimal tariff γ^* & system performance (f^*) to CPT model parametrization errors $(\vec{\theta} \neq \vec{\theta}_{true})$?

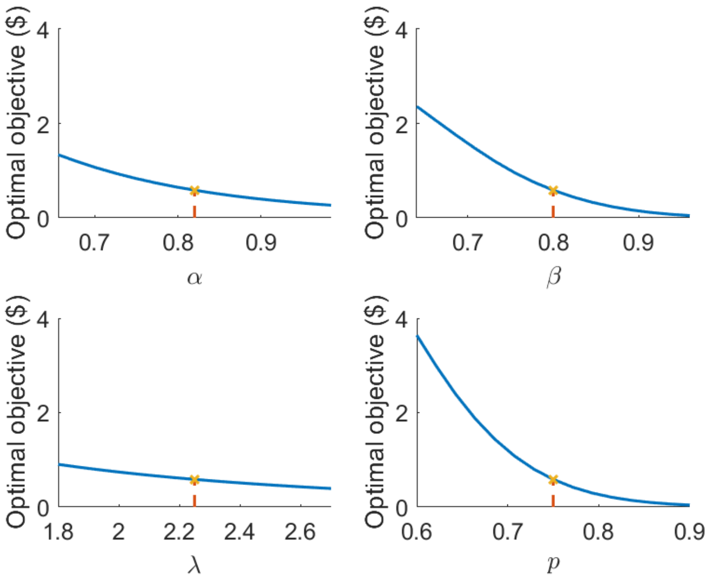
[18] C. Büskens and H. Maurer, “Sensitivity Analysis and Real-Time Optimization of Parametric Nonlinear Programming Problems,” in Online Optimization of Large Scale Systems, M. Grötschel, S. O. Krumke, and J. Rambau, Eds. Berlin, Heidelberg: Springer, 2001, pp. 3–16.

Sensitivity & robustness analysis results

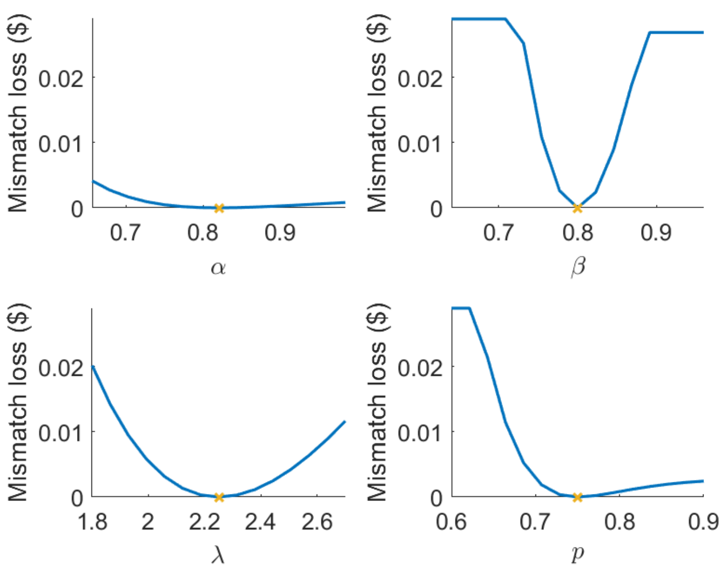
Optimal (revenue maximizing) tariff γ^*



Expected revenue $f^* = \gamma^* p_R^s(\gamma^*; \vec{\theta})$



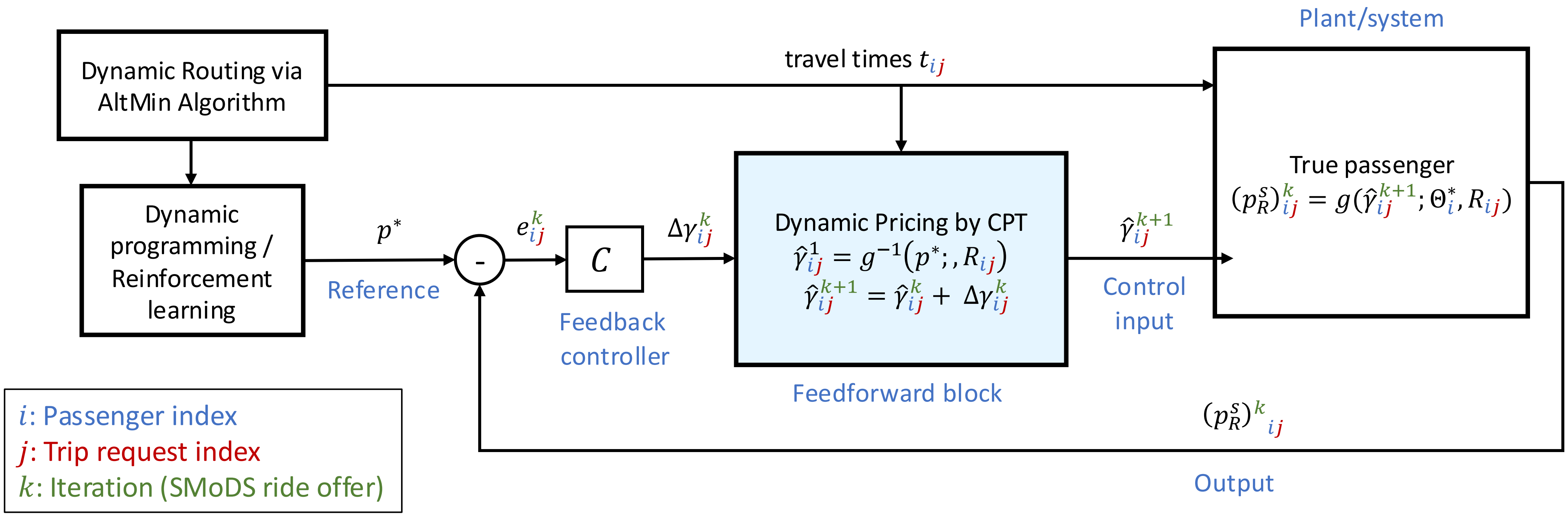
Mismatch loss $\Delta f = f(\gamma_{true}^*; \vec{\theta}_{true}) - f^*$



- Set $\alpha^+ = \alpha^- = \alpha$, $\beta^+ = \beta^- = \beta$
- Assume $R = u_2$ $(> u_1)$

SMoDS using CPT – Step 6: Closed-loop Transactive Control

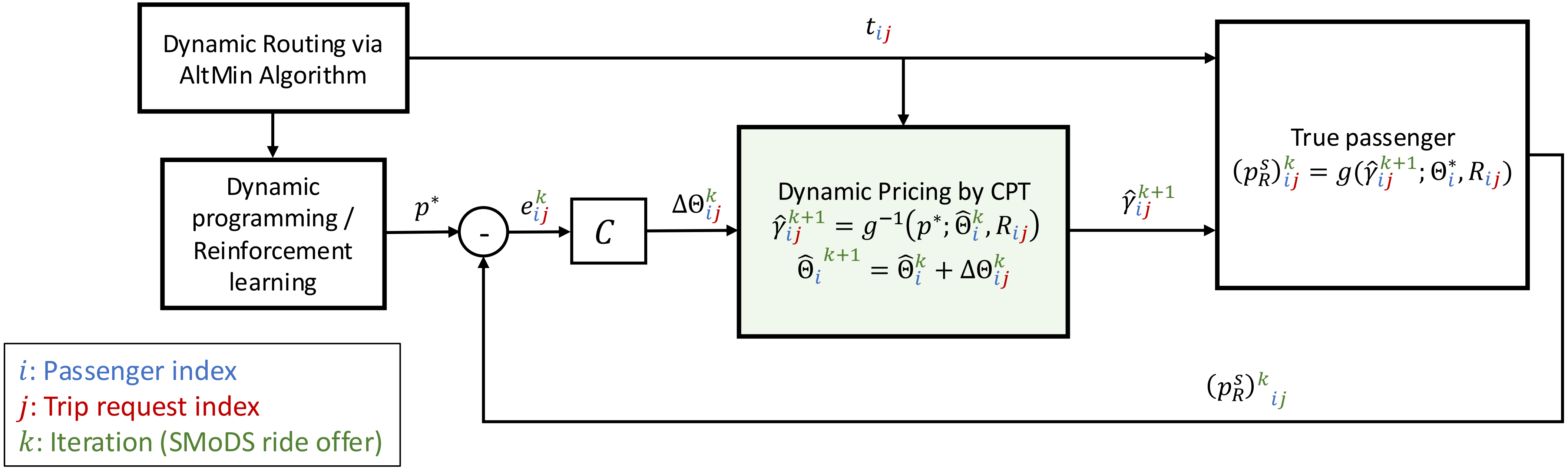
Approach I: Direct tariff-based



Goal: $p_R^s \to p^*(t)$ (averaged over a region)

Towards closed-loop transactive control (II)

Approach II: Indirect parameter-based



Goal: $p_R^S \rightarrow p^*(t)$ (averaged over a region) and $\widehat{\Theta}_i \rightarrow \Theta_i^*(t)\ \forall\ i$

Gradient descent based feedback controller

$$\min_{\hat{\gamma},\widehat{\Theta}} \quad L\left(\hat{\gamma},\widehat{\Theta}\right) = \frac{1}{2}\left\|\mathbf{p}_R^s(\hat{\gamma};\widehat{\Theta}) - \mathbf{p}^*\right\|_2^2$$

$$\text{s.t.} \quad \underline{\gamma} \leq \hat{\gamma} \leq \overline{\gamma}$$

$$\underline{\widehat{\Theta}} \leq \widehat{\Theta} \leq \overline{\widehat{\Theta}}$$

Approach I

$$\hat{\gamma}^{k+1} = \hat{\gamma}^k - \eta_k \left.\frac{\partial L}{\partial \gamma}\right|_{\hat{\gamma}^k}$$

$$\frac{\partial L}{\partial \gamma} = \left(\mathbf{p}_R^s - \mathbf{p}^*\right)\nabla_\gamma \mathbf{p}_R^s$$

$$\left.\nabla_\gamma \mathbf{p}_R^s\right|_k \approx \left.\frac{\Delta \mathbf{p}_R^s}{\Delta \gamma}\right|_{\hat{\gamma}^k} = \frac{(\mathbf{p}_R^s)^k - (\mathbf{p}_R^s)^{k-1}}{\hat{\gamma}^k - \hat{\gamma}^{k-1}}$$

Approach II

$$\hat{\mathbf{\Theta}}^{k+1} = \hat{\mathbf{\Theta}}^k - \eta_k \left.\frac{\partial L}{\partial \mathbf{\Theta}}\right|_{\hat{\mathbf{\Theta}}^k}$$

$$\left.\frac{\partial L}{\partial \mathbf{\Theta}}\right|_{\hat{\mathbf{\Theta}}^k} = \left(\mathbf{p}_R^s - \mathbf{p}^*\right)\nabla_{\mathbf{\Theta}} \mathbf{p}_R^s$$

$$\nabla_{\mathbf{\Theta}} \mathbf{p}_R^s = \frac{(\mathbf{p}_R^s)^k - (\mathbf{p}_R^s)^{k-1}}{\hat{\mathbf{\Theta}}^k - \hat{\mathbf{\Theta}}^{k-1}}$$

Learning rate

$$\Delta_k = \alpha \Delta_{k-1} + \eta \nabla_\theta L(\theta_{k-1})$$

$$\theta_k = \theta_{k-1} - \Delta_k$$

$$\eta_k = \eta_0 e^{-dk}$$

$$\eta_k = \frac{\eta_0}{1+dk}$$

Exponential decay

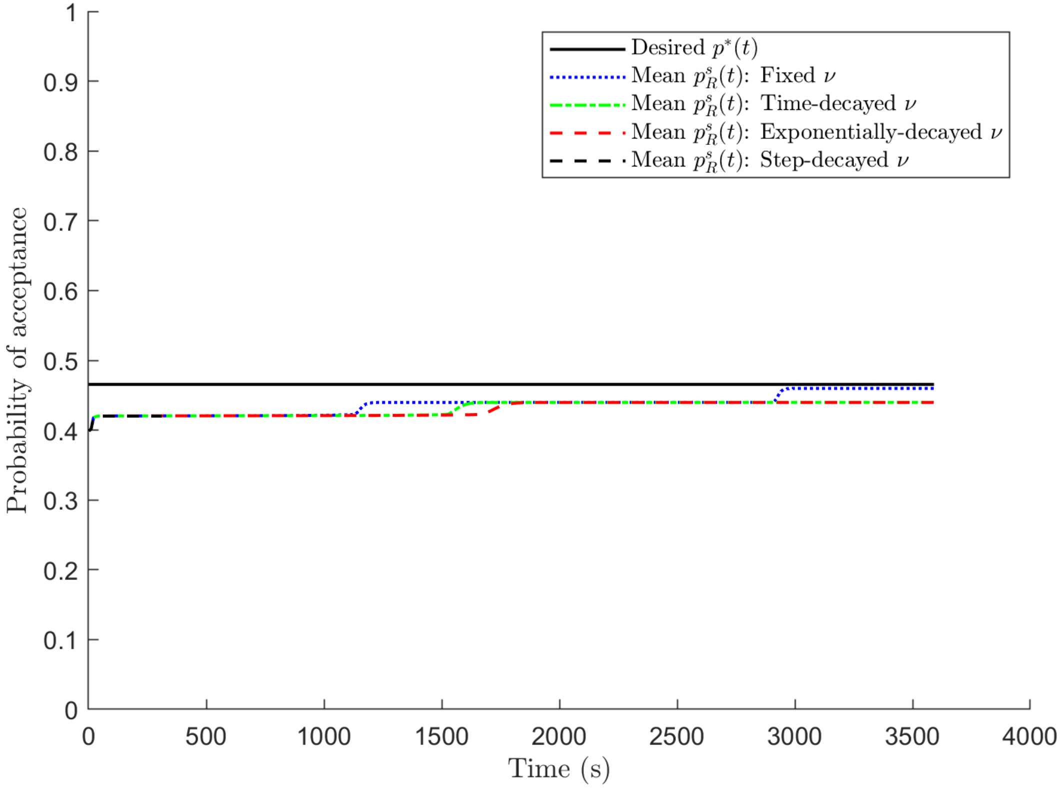
Adaptive (momentum)

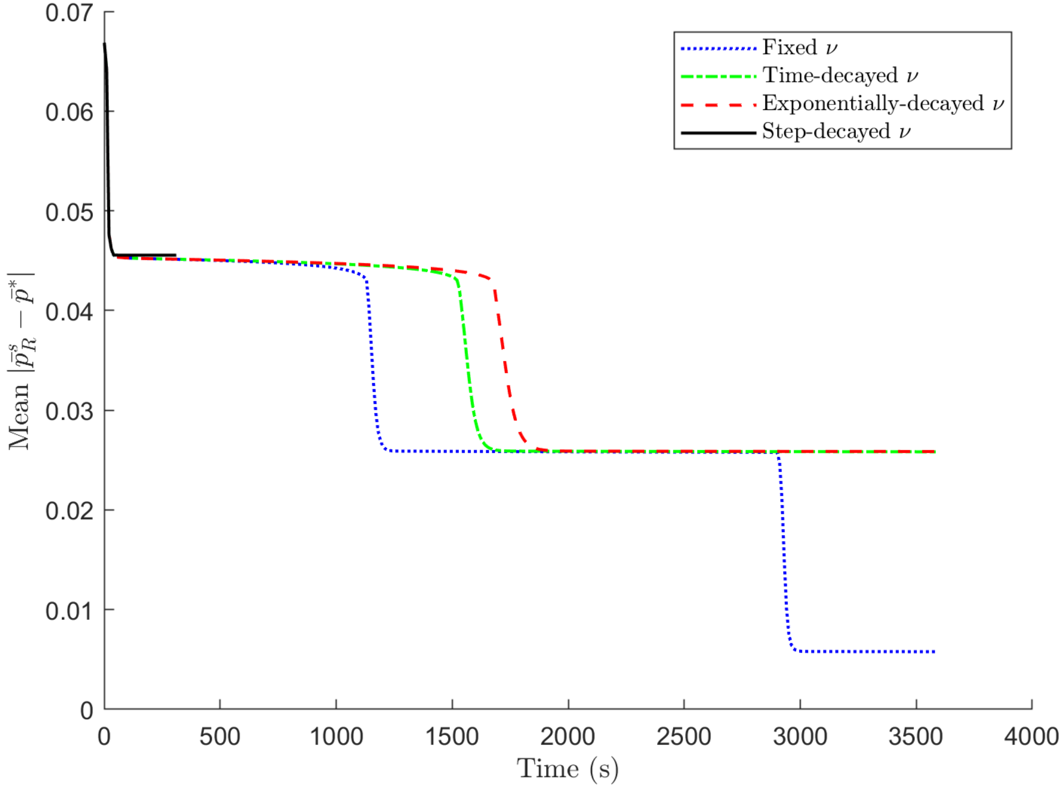
Step decay schedule

schedule

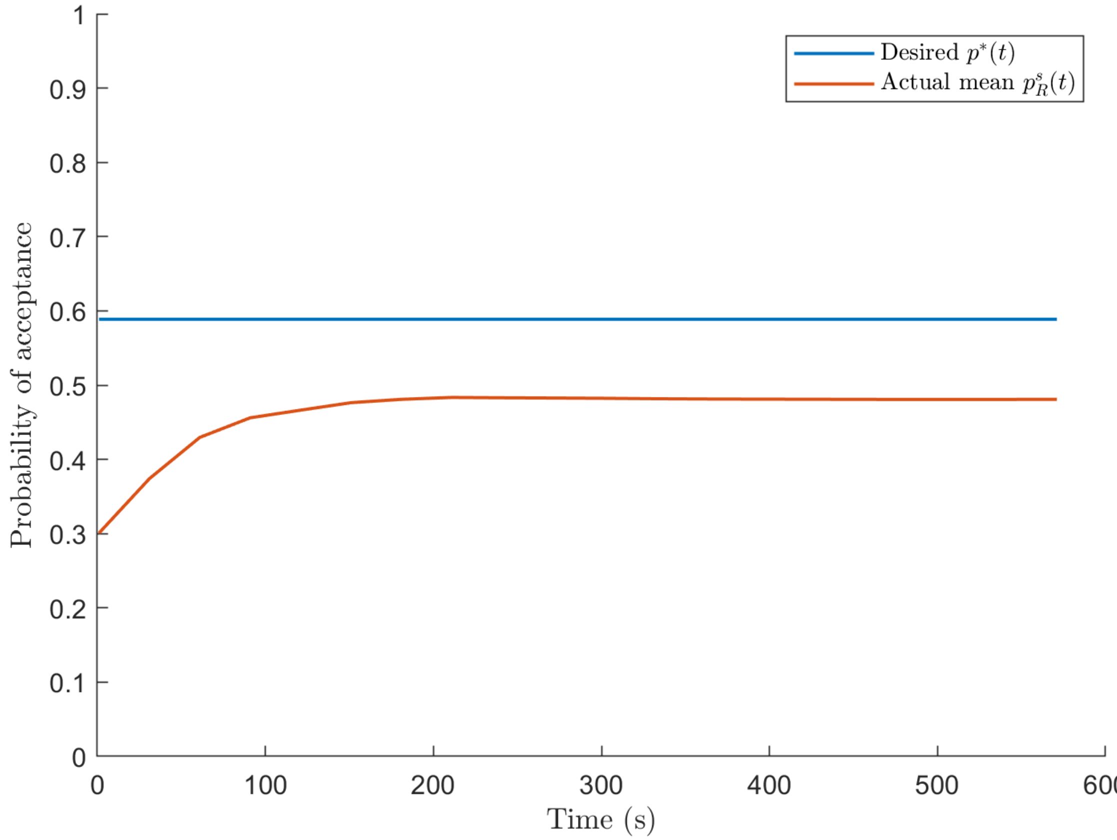
Preliminary results: Effects of learning rate

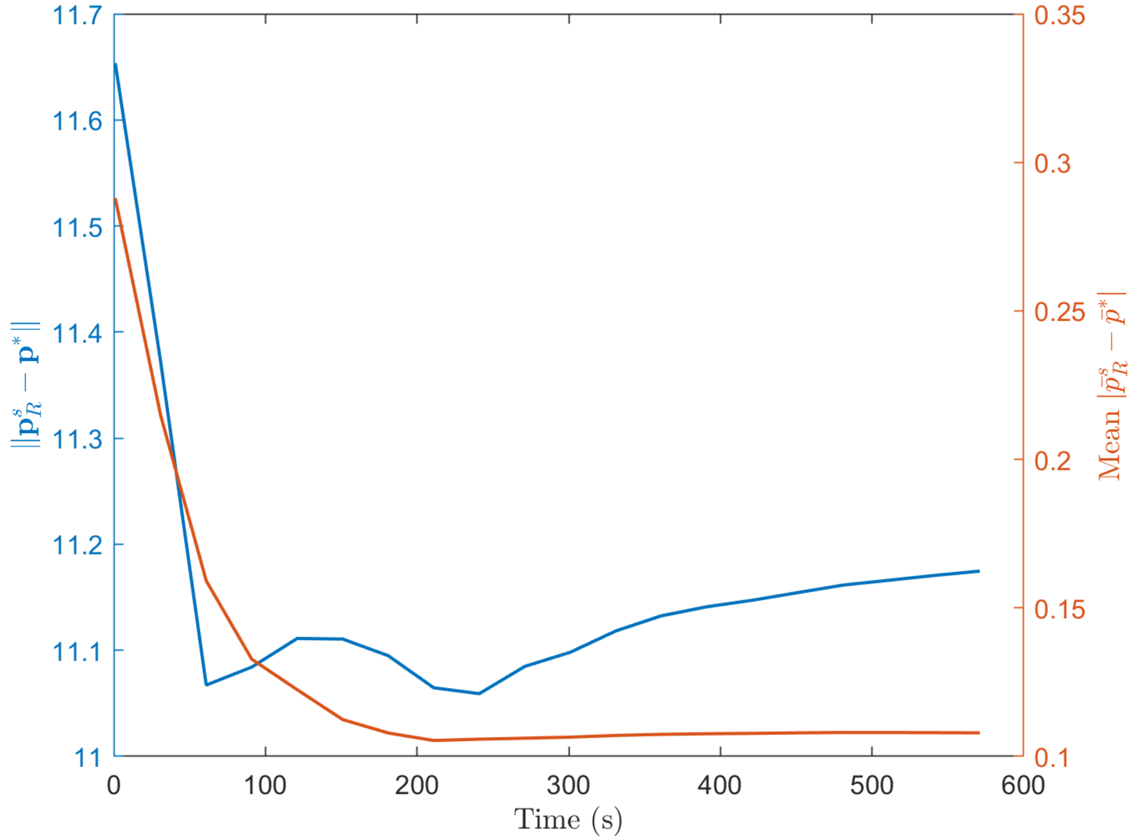
Using approach I: $\eta = 0.3, \mathrm{d} = 0.005$, step decay after every 5 iterations





Preliminary results: Pending challenges





Using adaptive learning rate with momentum, $\alpha = 0.8, \eta = 0.1$

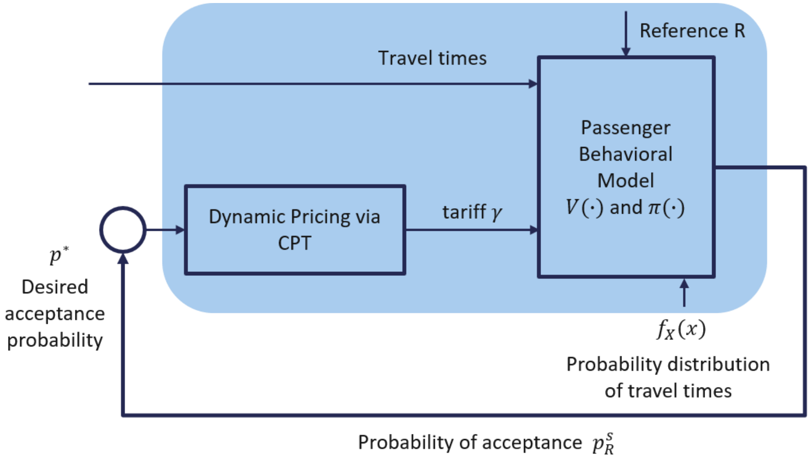
My contribution: CPT-based modeling and price control for SMoDS

Survey design
- Novel survey to elicit travel time risk preferences

Model estimation
- Validated mode choice model
- Extracted useful CPT risk parameters
- Compared travel vs financial risk attitudes

Sensitivity analysis
- Quantified CPT model's robustness to errors in passenger behavioral model
- Used results to inform closed loop control strategy

Transactive control
- Designed closed loop system to optimize SMoDS performance
- Developed two gradient descent based feedback control methods



*Developed **model-based** control for **cyber-physical-human** systems involving **risk & uncertainty***

1. **Nair, V. J.**, Guan, Y., Annaswamy, A. M., Tseng, H. E., & Singh, B., "Sensitivity Analysis of Passenger Behavioral Model for Dynamic Pricing of Shared Mobility on Demand", *Transportation Research Part A: Policy & Practice*, 2021 *(under review)*.

2. **Nair, V. J.**, Annaswamy, A. M., Tseng, H. E., & Singh, B., "Estimation of CPT behavioral model for dynamic pricing & transactive control of shared mobility on demand", *Transportation Research Part B: Methodological*, 2021 *(in preparation)*.

APPENDIX

Summary of model parameters

Parameter	Description
Coefficients and distributions for each travel alternative i	
$a_{walk}, a_{wait}, a_{ride,i} \leq 0$	Disutility due to walking, waiting and riding times
$b \leq 0$	Disutility due to tariff
ASC_i	Alternative specific constants (effects of all factors other than time and price)
Passenger risk attitudes (CPT)	
R	Reference
$\lambda > 1$	Loss aversion parameter
$0 < \beta^+, \beta^- < 1$	Diminishing sensitivity parameter
$0 < \alpha < 1$	Probability distortion parameter

Literature: Discrete choice analysis

- Discrete choice experiment surveys to estimate utility functions [3]
  – Stated preferences [4] > revealed preferences
  – Multinomial logit models: Fixed and random coefficients
- Mixed logit > Standard logit
  – Flexibility in underlying parameter distributions [5]
  – Variations in preferences across individuals [6]
  – Panel data with repeated choices [7]

[3] K. Train, Discrete Choice Methods with Simulation. Jun. 2009.
[4] J. J. Louviere, D. A. Hensher, and J. D. Swait, “Stated Choice Methods: Analysis and Applications,” Sep. 2009.
[5] K. Train, “Mixed logit with a flexible mixing distribution,” Journal of choice modelling, vol. 19, pp. 40–53, Jun. 2016.
[6] K. E. Train, “Recreation Demand Models with Taste Differences over People,” Land Economics, vol. 74, no. 2, p. 230, May 1998.
[7] D. Revelt and K. Train, “Mixed Logit with Repeated Choices: Households’ Choices of Appliance Efficiency Level,” Review of Economics and Statistics, vol. 80, no. 4, pp. 647–657, Nov. 1998.

Literature : Mode choice modelling

- Stated preference applications in transportation [8]
  - Forecast travel demand patterns
  - Value of time (VOT) and value of reliability (VOR) [9]
- Recent work in newer mobility services
  - Autonomous [10], electric & hybrid vehicles [11]
  - Ride hailing [12] and ride pooling [13]

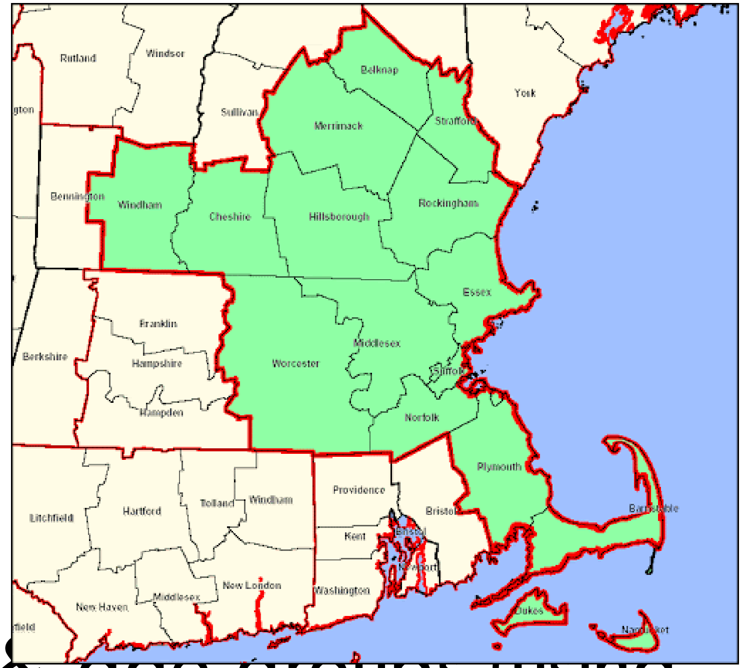
[8] M. E. Ben-Akiva and S. R. Lerman, Discrete choice analysis: theory and application to travel demand, ser. MIT Press series in transportation studies. Cambridge, Mass: MIT Press, 1985, no. 9.
[9] T. C. Lam and K. A. Small, "The value of time and reliability: measurement from a value pricing experiment," Transportation Research Part E: Logistics and Transportation Review, vol. 37, no. 2-3, pp. 231–251, Apr. 2001.
[10] R. Krueger, T. H. Rashidi, and J. M. Rose, "Preferences for shared autonomous vehicles," Transportation Research Part C: Emerging Technologies, vol. 69, pp. 343–355, Aug. 2016.
[11] G. Ewing and E. Sarigöllü, "Assessing Consumer Preferences for Clean-Fuel Vehicles: A Discrete Choice Experiment," Journal of Public Policy & Marketing, vol. 19, no. 1, pp. 106–118, Apr. 2000.
[12] M. Montazery and N. Wilson, "A New Approach for Learning User Preferences for a Ridesharing Application," in Transactions on Computational Collective Intelligence XXVIII, 2018, pp. 1–24.
[13] S. Naumov and D. R. Keith, "Hailing Rides Using On-Demand Mobility Platforms: What Motivates Consumers to Choose Pooling?" Proceedings, vol. 2019, no. 1, p. 19670, Aug. 2019.

Literature: Risk preferences with CPT

- Financial applications
  - Lotteries [14], insurance, investor portfolio selection [15]
  - Linking risk attitudes towards time & money [16]
- Transport applications
  - Route choice behaviors on highways [17]
  - Autonomous vehicles adoption [18]

[14] M. O. Rieger, M. Wang, and T. Hens, "Risk Preferences Around the World," Management Science, vol. 61, no. 3, pp. 637–648, Feb. 2014.
[15] C. Bernard and M. Ghossoub, "Static portfolio choice under Cumulative Prospect Theory," Math Finan Econ, vol. 2, no. 4, pp. 277–306, Mar. 2010.
[16] T. Tanaka, C. F. Camerer, and Q. Nguyen, "Risk and Time Preferences: Linking Experimental and Household Survey Data from Vietnam," American Economic Review, vol. 100, no. 1, pp. 557–571, Mar. 2010.
[17] R.-C. Jou and K.-H. Chen, "An application of cumulative prospect theory to freeway drivers' route choice behaviours," Transportation Research Part A: Policy and Practice, vol. 49, pp. 123–131, Mar. 2013.
[18] S. Wang and J. Zhao, "Risk preference and adoption of autonomous vehicles," Transportation Research Part A: Policy and Practice, vol. 126, pp. 215–229, Aug. 2019.

Survey eligibility criteria



- Restricted to Boston DMA (~4.8 million)
- > 18 years of age
- Valid driver's license
- Must complete survey on desktop
- Must have used ridesharing services before
- Nationally representative sample by gender & age groups (using census data)
- Total of 955 useful responses

Statistical significance of mode choice model

Asymptotic Student's t-test

$$t = \frac{\hat{\mu} - \mu_0}{SE} = \frac{\hat{\mu}}{SE}$$

Z-test

$$Z = \frac{\hat{\mu} - \mu_0}{s} = \frac{\hat{\mu}}{\sigma / \sqrt{n}}$$

Parameter mean	t-stat	p-value	Z-score	p-value
a_{walk}	-11.0141	0.0000	12.8238	2.000
a_{wait}	0.6244	0.7338	2.3518	1.9813
$a_{ride,\ transit}$	-7.9241	0.0000	-11.4393	0.0000
$a_{ride,\ exclusive}$	-6.1996	0.0000	45.7569	2.0000
$a_{ride,\ pooled}$	-14.2325	0.0000	60.7051	2.0000
b	-10.4452	0.0000	-26.8046	0.0000
$ASC_{exclusive}$	-14.4046	0.0000	-34.7825	0.0000
ASC_{pooled}	-14.8511	0.0000	-37.7979	0.0000

Effects of MSL hyperparameters

Example of CPT nonlinearities

Detection of CPT effects

Tuning regularization hyperparameters

Scatter plots of financial CPT parameters

Summary statistics of CPT parameters

Financial (lotteries)

	α^+	α^-	β^+	β^-	λ	Squared error norm
Mean	0.4456	0.1315	0.2166	0.3550	20.0494	0.8625
Median	0.4124	0.1320	0.2188	0.3649	11.8715	0.8439
SD	0.1828	0.0448	0.0985	0.1906	25.8554	0.3605

Travel (SMoDS)

	α^+	α^-	β^+	β^-	λ	Squared error norm (adjusted)
Mean	0.4390	0.4653	0.1721	0.6664	5.3998	3.4915
Median	0.4607	0.5054	10^{-5}	0.6930	1.0000	3.4599
SD	0.1333	0.1130	0.2438	0.1922	11.4421	1.1016

Sensitivity analysis derivations

Lagrangian dual

$$\mathcal{L}(\gamma;\vec{\theta}) = -f(\gamma;\vec{\theta}) + \mu_1 \cdot (\gamma - \overline{\gamma}) + \mu_2 \cdot (\underline{\gamma} - \gamma)$$

Karush-Kuhn-Tucker (KKT) conditions

1^{st} order necessary conditions

$$\frac{\partial \mathcal{L}}{\partial \gamma} = -f_\gamma(\gamma^*;\vec{\theta}) + \mu_1^* - \mu_2^* = 0$$

Complementary slackness conditions

$$\mu_1^* \cdot (\gamma^* - \overline{\gamma}) = 0\ ,\ \mu_2^* \cdot (\underline{\gamma} - \gamma^*) = 0$$

Primal problem feasibility

$$\mu_1^*\ ,\mu_2^* \geq 0$$

Strong 2nd Order Sufficient Condition

$$\nu^\intercal\, \mathcal{L}_{\gamma\gamma}\, \nu > 0 \quad \forall\, \nu \neq 0 \quad \text{s.t.} \quad g_\gamma^1\, \nu = 0 \quad \text{or} \quad g_\gamma^2\, \nu = 0$$

$$\begin{bmatrix} \frac{d\gamma^*}{d\theta} \\ \frac{d\mu^a}{d\theta} \end{bmatrix} = -\begin{bmatrix} \mathcal{L}_{\gamma\gamma} & g^{a\intercal} \\ g^a & 0 \end{bmatrix}^{-1} \begin{bmatrix} \mathcal{L}_{\gamma\theta} \\ g^a \end{bmatrix}$$

Sensitivity analysis derivations

If neither upper nor lower bound constraint active

$$\frac{d\gamma^*}{d\theta} = -\mathcal{L}_{\gamma\gamma}^{-1}\mathcal{L}_{\gamma\theta} \qquad \frac{df^*}{d\theta}(\gamma(\theta);\theta)|_{\theta=\theta_0} = \mathcal{L}_\theta(\gamma_0^*, \mu_0^a, \theta_0)$$

Taylor approx. for perturbed solutions

$$\gamma^*(\theta) = \gamma_0^* + \frac{d\gamma^*}{d\theta}(\theta_0)(\theta - \theta_0)$$

$$f^*(\theta) = f_0^* + \frac{df^*}{d\theta}(\theta_0)(\theta - \theta_0)$$

Local sensitivity analysis domain

$$\Delta\theta_{max} = -\frac{\mu^a(\theta_0)}{\frac{d\mu^a}{d\theta}(\theta_0)} \quad \text{OR} \quad \Delta\theta_{max} = -\frac{g_{ina}(\gamma_0;\theta_0)}{\frac{dg_{ina}}{d\theta}(\gamma_0;\theta_0)}$$

If changes in active constraint set detected, perform *global* sensitivity analysis instead

Mismatch loss

$$\Delta f = f(\gamma_{true}^*;\theta_{true}) - f(\tilde{\gamma}^*;\theta_{true})$$

$$\gamma_{true}^* = \underset{\gamma}{\operatorname{argmax}}\ f(\gamma;\theta_{true})$$

$$\tilde{\gamma}^* = \gamma^*(\tilde{\theta}) = \underset{\gamma}{\operatorname{argmax}}\ f(\gamma;\tilde{\theta})$$

[22] C. Büskens and H. Maurer, "Sensitivity Analysis and Real-Time Optimization of Parametric Nonlinear Programming Problems," in Online Optimization of Large Scale Systems, M. Grötschel, S. O. Krumke, and J. Rambau, Eds. Berlin, Heidelberg: Springer, 2001, pp. 3–16.