Voltage Regulation in Distribution Grids: A Survey

Priyank Srivastava[a], Rabab Haider[a], Vineet J. Nair[a], Venkatesh Venkataramanan[b], Anuradha M. Annaswamy[a], Anurag K. Srivastava[c]

[a] *Department of Mechanical Engineering, Massachusetts Institute of Technology*
[b] *National Renewable Energy Laboratory*
[c] *Lane Department of Computer Science and Electrical Engineering, West Virginia University*

Abstract

Environmental and sustainability concerns have caused a recent surge in the penetration of distributed energy resources into the power grid. This may lead to voltage violations in the distribution systems making voltage regulation more relevant than ever. Owing to this and rapid advancements in sensing, communication, and computation technologies, the literature on voltage control techniques is growing at a rapid pace in distribution networks. In particular, there is a paradigm shift from traditional offline centralized approaches to distributed ones leveraging increased and varied types of actuators, real-time sensing, fast and efficient computations, and an overall distributed situational awareness. This paper reviews state-of-the-art voltage control algorithms, summarizes the underlying methods, and classifies their coordination mechanisms into local, centralized, distributed, and decentralized. The underlying solution methodologies are further classified into two categories, open-loop and feedback-based. Two specific examples are provided to illustrate these solutions for voltage regulation.

1. Introduction

Secure and reliable operation of the power system requires the system frequency and voltages to be within certain acceptable limits. The increasing penetration of renewable energy resources and their intermittent nature results in voltage fluctuations throughout the grid and is making this task increasingly challenging. Consequently, to make sure voltage stability is not the bottleneck in achieving net-zero, it is imperative to develop and implement new technologies for voltage control.

Traditionally, voltage control in the distribution grid has been implemented using devices such as tap changing transformers, shunt capacitors, and voltage regulators. However, these electro-mechanical devices were not designed to handle the new level of variability that comes with high penetration of intermittent distributed energy resources (DERs) such as solar photovoltaic (PV). Efforts to maintain voltages within acceptable operating range under such high variability can cause devices to actuate frequently, which is not desirable as the lifespan can decrease drastically (see, for example, McDonald (2013)). Dynamic Volt-Ampere Reactive (VAR) devices such as smart inverters are an attractive addition for achieving voltage control. As DERs such as PVs are often accompanied by smart inverters, the proliferation of DERs enables many more opportunities for achieving voltage control. These devices can provide low-cost and fast timescale reactive power compensation throughout the distribution grid, reducing the mechanical switching burden on traditional devices and improving voltages in grids with high intermittent generation. In addition, these DERs are typically equipped with local intelligence and computational abilities, not only in terms of VAR devices but also home energy management systems that optimize local storage devices, dynamic charging of electric vehicles, or smart thermostats consumer constraints and preferences. This increasing and varied presence of actuators with smart sensing, efficient processing and fast computation, with all of which occurring in a distributed manner, provides great opportunities for voltage control, especially in distribution grids. Traditional methods, which were purely reactive and responsive, can now be replaced with increasingly 'smart' strategies which are predictive, prescriptive, and increasingly autonomous. In general, this enables 'smart' grids of the future to adopt widespread automation for data acquisition, grid operations, and decision making. Our focus in this paper is on voltage control, an important problem encouraged by the overall trend of automated decision making in power grids.

The traditional voltage control architecture in the distribution grid consists only of primary control employing reactive power-voltage (Q-V) droop curves and proportional-integral (PI) control, typically at the local device level or centrally by system operators. Adoption of DERs throughout the grid provides the opportunity to coordinate across multiple devices (rather than local decision making) and the use of secondary control which focuses on optimiz-

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ing grid voltages. However, harnessing the true capabilities of these DERs requires that we move away from the standard practice of system operators in control centers overseeing the decision making to an automatic coordination of hundreds or thousands of distributed devices at fast timescales. This is evident from the growing body of works on decentralized and distributed voltage control algorithms and in addition supported by cost declines due to the maturation of enabling technologies (e.g., communication and distributed computation units). All of this makes it timely to carry out a survey of this dynamic area of voltage control, the focus of this paper.

In this paper, we focus our attention on recent works on voltage control at the distribution network level. An important contribution of this paper compared to previous survey papers on voltage control (e.g., Rebours et al. (2007a,b); Mahmud and Zahedi (2016); Antoniadou-Plytaria et al. (2017); Sun et al. (2019); Fusco et al. (2021)), is the detailed discussion about how the sensory data is deployed in updating the algorithms. In addition to classifying various papers on the basis of network models, coordination mechanisms, heuristic and theoretical methods, objectives, and actuation, we also categorize them on the basis of the solution methodology, which may be an open-loop or a feedback-based (closed-loop) approach. We restrict our attention to time-scales that correspond to optimization; we defer the reader to works such as Miranbeigi et al. (2019); Colombino et al. (2017) for methods that involve control that accommodates time-scales of power electronics. We do not focus on methods that are model-free (such as the ones described briefly in Section 4.3, many of which use reinforcement learning techniques) but rather those that employ a power systems based model. Overall, the contributions of this paper are as follows:

- Comprehensive taxonomy of the reviewed papers captured succinctly in tables.
- Classification based on solution methodologies that may be either open-loop or feedback-based.
- Identification of key barriers in the practical implementation of the current research.

This paper is organized as follows. In Section 2, we explain how voltage regulation could be formulated naturally as an optimization problem. In Section 3, we classify the reviewed papers in Table 1 and Table 2, and provide a brief description of different power network models used, coordination mechanisms employed, heuristic and theoretical methods, actuation devices, objectives considered, and solution methodologies. In Section 4, we point out the gaps in the current literature and highlight key issues that warrant further research attention. These include concerns about load modeling, validation testbeds, performance guarantees for model-free methods, communication protocols, cybersecurity considerations, industrial practice, and resilience. Finally, Section 5 concludes the paper.

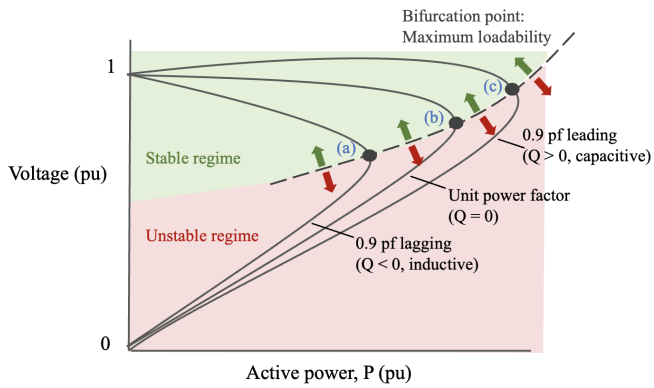


Figure 1: Nose curves demonstrating the critical active power values for different values of the reactive power.

2. Voltage Regulation Optimization Problem

The voltage regulation problem can be cast as an optimization of the form:

$$\begin{aligned} &\min_{x} \; f(x) \\ &\text{s.t.} \begin{cases} h(x) = 0 \\ g(x) \leq 0 \end{cases} \end{aligned} \tag{1}$$

where the decision variable x is usually the reactive power setpoints of the DERs (which is of utmost importance in stabilizing the system voltages as explained in this section below), but could also include the active power setpoints, and the tap settings of transformers, the latter giving rise to mixed integer problems. The objective function f describes system-wide objectives such as real power losses, congestion of real flows, and maintaining adequate voltages throughout the system. The first of these, of loss minimization, is a typical objective in distribution grids since roughly 40% of total power system losses occur at the distribution level (see Section 3.5 for a more detailed discussion). The equality constraints h come from the power flow equations (see Section 3.1 for details). The inequality constraints g represent the voltage limits, constraints on actuators, and flow limits of the lines. As DERs are becoming increasingly common in distribution grids, these actuator constraints can include active and reactive power injection limits of smart inverter-based resources (IBRs), charge or discharge limits of battery devices, or flexibility range of demand response loads for example.

Voltage Regulation and Reactive Power

While active power (P) runs the motor, illuminates the bulb, and heats the home, reactive power (Q) is needed to support system voltages and enable active power transfer. *If the network voltages are too low, real power cannot be supplied; reactive power is needed to provide the voltage levels necessary for real power transfer.* Figure 1 shows the P-V curve (often called the 'nose' curve) for different power factors. For a typical network, the power factor will be lagging and reactive power will be drawn from the network by the loads. This constrains the active power that can be delivered to loads without violating the voltage stability, denoted on the figure as (a) on the maximum loadabilty curve connecting the voltage bifurcation points. This in turn implies that voltage regulation involves injections of reactive power (capacitive or leading) into the network so that the active power transferred can be increased and we can go from (a) to (b) or (c) indicated in Figure 1. The presence of DERs, equipped with inverters that can inject reactive power into the grid, expands the ability to achieve voltage regulation, as DERs increase the number of voltage regulating devices that can be used to support grid voltages, and facilitate the move from (a) to (b) to (c). This is particularly effective as reactive power travels poorly, as requiring reactive power to be carried across the transmission network to a load center will increase line currents, incur higher losses, and reduce the real power carrying capacity of lines. All of these imply that the possibility of accomplishing effective voltage regulation in a distribution grid, through carefully coordinated control of DERs that are present in distributed locations, is an important problem and forms the focus of this survey paper.

3. Classification of the Papers in this Survey

We classify the papers related to voltage regulation based on the following features: (i) the power system model deployed depending on the underlying assumptions, (ii) coordination mechanism used based on the communication and computation pattern, (iii) heuristic or theoretical method employed, (iv) actuation or control inputs used, and (v) objective function considered. Table 1 presents a summary of the methods with all these features listed. Of particular interest to the control community is the type of coordination mechanisms (column 3), which may be local, centralized, decentralized, or distributed. While the first three are discussed in Table 1, we relegate the papers based on distributed methods, on which much of recent methods have focused on, to Table 2, with additional classification based on the underlying solution methodology. In particular, the distributed methods are classified into open-loop and feedback-based methods. In what follows, we discuss papers based on each of the features mentioned in Tables 1 and 2.

3.1. Power Network Models

The underlying physics of the grid are captured in the alternating current (AC) power flow equations. Here, we introduce the power flow equations following (Grainger and Stevenson, 1994; Kundur, 1994). Consider a power distribution network with n buses. The active and reactive power injections at bus $i \in \{1, \ldots, n\}$, denoted as P_i and Q_i, respectively, are given by

$$P_i = \sum_{k=1}^{n} |Y_{ik} V_i V_k| \cos(\theta_{ik} + \delta_k - \delta_i) \tag{2a}$$

$$Q_i = -\sum_{k=1}^{n} |Y_{ik} V_i V_k| \sin(\theta_{ik} + \delta_k - \delta_i) \tag{2b}$$

where $Y_{ij} = |Y_{ij}| \angle \theta_{ij}$ is the complex admittance between buses i and j, $\{V_j\}_{j=1}^n$ and $\{\delta_j\}_{j=1}^n$ denote the voltage and angle at bus $j \in \{1, \ldots, n\}$, respectively. P_i and Q_i are positive if bus i is injecting power into the system and negative if it absorbing power from the system. Equation (2) is known as the *branch flow model* of the power balance equation. Another commonly used model which uses nodal power injections and voltages is given by

$$\mathbf{p} + j\mathbf{q} = \mathrm{Re}\{\mathrm{diag}(\mathbf{v}\mathbf{v}^* \mathbf{Y}^*)\} + j\mathrm{Im}\{\mathrm{diag}(\mathbf{v}\mathbf{v}^* \mathbf{Y}^*)\} \tag{3}$$

where $(\cdot)^*$ denotes the complex conjugate, $\mathbf{p}$, $\mathbf{q}$, $\mathbf{v}$ denote the vectors containing the entries of $\{P_i\}_{i=1}^n$, $\{Q_i\}_{i=1}^n$, and $\{V_i\}_{i=1}^n$, respectively, and $\mathbf{Y}$ denotes the bus admittance matrix whose $ik-$th element is given by Y_{ik}. Equation (3) is known as the *bus injection* model. Both the branch flow (2) and bus injection models (3) are equivalent (Subhonmesh et al., 2012) and any solution to the voltage regulation optimization problem should satisfy (2) or(3) (encoded via $h(x)$ in (1)) for it to be practically feasible. However, both (2) and (3), which correspond to AC power flow equations, are highly nonlinear, making the optimization problem non-convex. As such, numerous relaxations have been proposed to deal with the complexity coming from the nonlinearity and nonconvexity of the power flow equations. Below we provide a brief description of simplified models commonly used in voltage regulation.

The works based on semidefinite programming (SDP)(Zhang et al., 2015; Robbins and Domínguez-García, 2016) reformulate (3) by defining a new variable, say $W = \mathbf{v}\mathbf{v}^*$ along with the constraint $\mathrm{rank}(W) = 1$. The non-convex rank constraint is then omitted to obtain a convex relaxation of (3). Another widespread approach is the second-order cone programming (SOCP) based relaxation, giving rise to the *DistFlow* model (Ding et al., 2017; Farivar et al., 2011; Li et al., 2020; Magnusson et al., 2019a). In this approach, a new variable

$$l_{ik} = \frac{P_{ik}^2 + Q_{ik}^2}{V_i} \tag{4}$$

is introduced for all $i, k \in n$. Here P_{ik} and Q_{ik} denote the active and reactive power transfer between buses i and k. l_{ik} denotes the square magnitude of the current flowing from i to k and the the power flow equation (2) could be expressed linearly with l_{ik}. However, the additional reformulation constraint (4) is non-linear making the optimization problem non-convex. To deal with this, in the DistFlow model, equality sign in (4) is replaced with a greater than or equal to ($\geq$), which makes the problem tractable and allows the determination of solutions that are close to optimal.

Table 1: Classification of local (**L**), centralized (**C**), and decentralized (**De**) voltage control algorithms. Two or more abbreviations in succession represent the hierarchical mechanisms, with each abbreviation corresponding to one layer in the hierarchy. **D** denotes the distributed methods.

Reference	**Power network model**	**(Coordination mechanism) + Heuristic or theoretical method**	**Actuation**	**Objective**
Ghosh et al. (2014)	Unbalanced 3-phase (`OpenDSS`)	(**L**) Droop-based (heuristic)	PV smart inverter P/Q	Keep V within limits
Capitanescu et al. (2014)	Nonlinear non-convex bus injection	(**C**) Mixed integer nonlinear program (MINLP)	DG P/Q, LTCs, switches, shunt banks	Minimize P curtailment, V within limits
Ding et al. (2017)	DistFlow (Branch flow)	(**C**) Mixed integer second order conic program	PV smart inverter P/Q	Minimize network losses, maximize PV generation output
Yeh et al. (2012)	LinDistFlow	(**C**) Multiobjective problem with switching law	PV inverter Q	Minimize V variations and network losses
Zafar et al. (2016)	Unbalanced (`DigSILENT PowerFactory`)	(**C**) MINLP, PSO	SVRs, BESS	Maximize BESS charging; minimize SVR tap operation, demand, losses; improve V profile
Cagnano and De Tuglie (2015)	Dynamic load flow (MATLAB Simulink)	(**C**) Lyapunov sensitivity method to find dynamical system equilibrium	PV inverter Q	Nodal V magnitude (reference/tracking error)
Kawano et al. (2016)	—	(**C**) Heuristic using voltage fluctuation forecasts via JIT-modeling	LTCs, SVRs	Maximize minimum V margin from limits
Xu et al. (2020a)	LinDistFlow (based on V sensitivities)	(**C**) Quadratic program (QP) optimization	DER P, Q	Minimize P/Q injection costs, V limit violations
Shen and Baran (2013)	Unbalanced nonlinear nonconvex 3-phase	(**C**) Iterative gradient-based optimization with forward/backward sweep	Q injections from solid state transformers	Minimize substation real power (aggregated customer load + losses)
Liu et al. (2019)	Unbalanced LinDistFlow	(**DL**) Partial primal-dual gradient updates using feedback	DER Q injection	Minimize weighted V mismatch, flat profile
Farivar et al. (2011)	DistFlow SOCP	(**C**) Hierarchical (fast-slow timescale) optimization	Slow: Shunt capacitors, LTCs; Fast: PV inverter Q injection	Minimize network losses, real power consumption, weighted sum of V magnitudes (CVR), LTC/capacitor switching costs
Valverde and Van Cutsem (2013)	Linearized (based on sensitivity matrices)	(**C**) QP Model predictive control (MPC)	DG P/Q outputs, LTC transformer V setpoint	Minimize changes of control variables while satisfying V limits
Arnold et al. (2016)	Unbalanced LinDistFlow	(**C**) QP optimization	Inverter Q injection	Minimize node-to-node V magnitude differences
Liu et al. (2017)	LinDistFlow	(**De**) QP optimization with gradient projection gradient descent for primal updates	DER Q	Minimize V mismatch error
Magnusson et al. (2019a)	LinDistFlow	(**De**/**D**)† Dual decomposition gradient update	Inverter Q injection	Maintain V within limits
Pachanapan et al. (2012)	Nonlinear Newton-Raphson load flow (`DigSilent PowerFactory`)	(**DeD**) Adaptive zone identification for power converter controllers using V sensitivities	DG, BESS power converters	Maintain V within limits
Mokhtari et al. (2013)	Single phase nonlinear Low Voltage (LV) (`PSCAD`)	(**DeDeL**) Heuristic, consensus-based	PV, BESS inverter P/Q	Mitigate V rise issues, equal P sharing among DERs

continued ...

†both variants proposed

Table 1: (continued ...) Classification of local (**L**), centralized (**C**), and decentralized (**De**) voltage control algorithms.

Reference	Power network model	(Coordination mechanism) + Heuristic or theoretical method	Actuation	Objective
Yorino et al. (2015)	Linearized model with V sensitivities	(**De**) Switching optimal control law	LTCs, step V regulators, tap-changing transformers	Minimize V deviations
Zhang et al. (2016)	Nonlinear power flow using Newton-Raphson, Gauss-Seidel, forward/backward sweep	(**De**) Agent-based finite state machine	Shunt capacitor switches, V regulator tap positions	Maintain V profile; minimize losses, switching of shunt capacitors
Ding et al. (2018)	Bus injection	(**De**) Multiobjective constrained nonlinear problem with PSO	Q absorption, P curtailment at controllable PV nodes	Minimize V fluctuations, network losses
Bahramipanah et al. (2016)	Nonlinear load flow	(**DeD**) Thevenin equivalent-based and recursive approaches	P/Q from BESS power converters	Nodal V setpoint tracking, BESS state of charge reference
Zhao et al. (2018a)	Bus injection	(**De**) PSO with sensitivity-based V update	Controllable PV units	Minimize PV Q absorption, P curtailment
Baker et al. (2018)	Linearized	(**L**) Convex optimization for proportional tuning of volt-VAR control (VVC) droop slopes	DER Q outputs	Minimize V profile deviations, actively participating DERs
Ding et al. (2020)	Fixed-point linearization based unbalanced	(**De**) Projected gradient-based convex optimization	Smart inverter P/Q	Voltage profile, CVR; Minimize P curtailment, Q output
Tasnim et al. (2023)	Load flow (MATLAB)	(**C**) Internal game-theory based algorithm	LTCs	Voltage profile, fast response, minimize LTC operations
Mansourlakouraj et al. (2021)	Piecewise-linearized	(**C**) Two-timescale risk-constrained optimization	Capacitor banks, PV and EV inverter power factor control	Minimize P loss
Wei et al. (2016)	Linearized	(**L**) Iterative, heuristic-based algorithm	DG P/Q outputs	Minimize voltage deviations, Q dispatch
Zhou et al. (2021)	LinDistFlow	(**L**) (i) Piecewise linear switched control law, (ii) Subgradient-based optimization	PV inverter Q injection	Minimize Q dispatch cost, V deviations from nominal
Singhal et al. (2019)	Unbalanced three-phase (`GridLAB-D`)	(**L**) Adaptive update of (i) Q based on V errors, (ii) droop slope based on V flickers	PV smart inverters	Minimize steady-state V setpoint error
Zhou et al. (2016a)	LinDistFlow	(**L**) Pseudo-subgradient based incremental updates	PV Q injections	Convex quadratic cost
Weckx and Driesen (2016)	Linearized (V sensitivity-based)	(**L**) QP to set optimal piecewise constant/linear Q(P) characteristics	PV Q injection/absorption	Minimize Q dispatch, keep V within limits
Shah and Crow (2016)	Unbalanced	(**L**) Incremental heuristic-based switching updates	Solid-state transformer Q injection/absorption	V profile within limits
Li et al. (2020)	DistFlow	(**L/D**)† Parameter tuning of piecewise linear Q-V curves via SOCP and MPC, ADMM for distributed version	DG Q	Minimize V deviations, power losses
Zhao et al. (2018b)	Linearized (First order Taylor series expansion)	(**L**) Mixed integer linear program (MILP) to optimize Q-V control curves	PV inverter Q compensation	Minimized V deviations

continued ...

†both variants proposed

Table 1: (continued ...) Classification of local (**L**), centralized (**C**), and decentralized (**De**) voltage control algorithms.

Reference	**Power network model**	**(Coordination mechanism) + Heuristic or theoretical method**	**Actuation**	**Objective**
Zhu and Liu (2016)	LinDistFlow	(**L**) Gradient-projection based method for box-constrained QP	PV Q injection	Minimize V mismatch norm, costs of supplying Q
Calderaro et al. (2011)	Linear (V sensitivity-based)	(**L**) Sensitivity-analysis based P, Q updates	P/Q from wind turbine power converters	Voltage profiles; minimize turbine disconnection, P curtailment
Farivar et al. (2013)	LinDistFlow	(**L**) Optimal parameters for switching piecewise linear VVC curves via convex optimization and dynamical system equilibrium-based approach to set	Inverter Q	Minimize V deviations from nominal, Q dispatch costs
Li et al. (2019)	DistFlow	(**L**) Kriging metamodel-based Q update	DG Q compensation	Minimize V deviations, network losses
Cuffe and Keane (2015)	Nonlinear (`DigSilent PowerFactory`)	(**L/C**)[†] Local: Multi-scenario Linear program (LP); Centralized: Single-period LP	Local: DG droop curve parameters, static V ratio of bulk transformer; Centralized: DG Q outputs, bulk transformer tap settings	Local: Maximize Q injection for under-V, Q absorption for over-V; Centralized: Minimize V deviations
Zhang et al. (2013)	Bus injection	(**L**) Iterative Q updates based on V errors	DER Q injections, LTCs, switched capacitors	Minimize V magnitude deviations from reference profile
Zhu and Li (2016)	LinDistFlow	(**L**) Box-constrained QP via gradient projection method	DER Q outputs	Flat V profile
Zhou et al. (2016b)	DistFlow	(**L**) Tune slopes of piecewise linear droop functions by finding equilibria of non-cooperative VVC game	PV inverter Q compensation	Bus V profiles, minimize Q costs
Li et al. (2014)	LinDistFlow	(**L**) QP via dual gradient method	Q compensation	Maintain V within limits, minimize Q injection costs
Bidgoli and Van Cutsem (2018)	Linearized via sensitivity matrices	(**CL**) Local: Q-V characteristic with corrections for over/under-V, Centralized: QP via MPC	DG P/Q outputs, LTC V setpoint	Minimize DG P/Q deviations
Nowak et al. (2020)	Full nonlinear (MATLAB/`OpenDSS`)	(**CL**) Local PI control + central ACOPF to set optimal V, Q setpoints for local controllers	DER Q control	Minimize delivery losses; maintain bus V near setpoints
Xu et al. (2022)	DistFlow with conic (SOCP) relaxation	(**CL**) Distributionally robust chance-constrained optimization	PV inverter P, Q	Minimize P import from upstream grid, P curtailment, losses;
Weckx et al. (2014)	Unbalanced 3-phase, 4-wire LV; linearized (V sensitivity-based)	(**De**) Langrangian dual decomposition	EV charging P, Q setpoints	Minimize EV charging costs, maximize customers' welfare, satisfy transformer capacity limits
Yu et al. (2012)	Meshed, linearized by sensitivity matrices	(**DeL**) ϵ-decomposition, LP	DG Q via power factor control, P curtailment	Maintain V within bounds
Robbins et al. (2013)	Linearized (sensitivity matrices) unbalanced	(**DL**) Switching control	DER Q injection	Prevent V limit violations
Valverde et al. (2019)	`OpenDSS`	(**LDDe**) Modified VVC curve	PV Q injection	V setpoint, maximize Q export from LV network

[†]both variants proposed

Table 2: Classification of distributed (**D**) voltage control algorithms.

Reference	Power network model	Heuristic or theoretical method	Solution Methodology	Actuation	Objective
Utkarsh et al. (2017)	Linearized balanced	Consensus-based dimension-distributed PSO	Open-loop	Q injection/ absorption by DGs, controllable loads	Minimize network P losses
Schiffer et al. (2014)	Nonlinear meshed	Consensus-based	Open-loop	Inverter Q	Proportional Q sharing
Bolognani and Zampieri (2013)	Linearized	QP optimization with randomized, leader-less, gossip-like algorithm	Open-loop	DGs Q	Minimize power distribution losses
Bolognani et al. (2013)	Linearized	QP dual ascent	Open-loop	DG Q compensation	Minimize line losses
Zhang et al. (2015)	SDP convex relaxation	Consensus-based	Open-loop	DER Q, DER/DR P	Minimize losses
Zhou et al. (2018)	Linearized	Stochastic dual ascent	Open-loop	PV inverters, thermostatically controllable loads (TCLs)	Minimize P curtailment, Q injection/absorption
Zhang et al. (2018)	Linearized	QP with ADMM	Feedback-based	Inverter P, Q setpoints	V profile; minimize P curtailment, Q dispatch
Magnússon et al. (2020)	Nonlinear DistFlow	QP with dual ascent	Feedback-based	DR P, PV inverter Q	Minimize P, Q generation costs
Bolognani et al. (2015)	Linearized	QP with dual ascent	Feedback-based	Q injection	V profile, minimize losses
Patari et al. (2021)	3-phase unbalanced LinDistFlow (`OpenDSS`)	QP with augmented Lagrangian-based primal-dual	feedback-based	DER Q injection	Nodal and network-level operating costs (for control action)
Liu et al. (2018)	Linearized 3-phase unbalanced	Consensus-based ADMM	Feedback-based	DER Q	Minimize V deviations, Q operating costs
Wu et al. (2017)	LinDistFlow	Primal-dual decomposition	Open-loop	PV inverter Q	Minimize V deviations, inverter operation costs
Tang et al. (2019)	LinDistFlow	QP with projected gradient dual descent	Open-loop	DER Q	V profile
Ortmann et al. (2020)	Linearized	Dual ascent	Feedback-based	Inverter Q	V profile
Zhou et al. (2017)	Nonlinear	Primal-dual projected gradient	Feedback-based	Customer P, Q setpoints	Maximize end-customers' social welfare, minimize network-level V deviations from nominal
Cortés and Martínez (2016)	Linearized	Primal-dual	Feedback-based	Q injections, V setpoints	Minimize cost of P imports from transmission grid, losses
Bolognani et al. (2019)	Linearized	Primal-dual decomposition with subgradient dual ascent	Feedback-based	Q injection/ absorption from DGs	V profile, minimize cost of Q
Todescato et al. (2018)	Linearized	Primal-dual	Feedback-based	Q injections	V magnitude profile
Qu and Li (2020)	LinDistFlow	Primal-dual gradient method with augmented Lagrangian	Open-loop	Q compensation	Minimizes losses, operating costs
Robbins and Domínguez-García (2016)	Unbalanced first-order linearized DistFlow	QP with ADMM	Open-loop	Q from controllable DERs	Minimize V magnitude deviations

continued ...

Table 2: (continued ...) Classification of distributed (**D**) voltage control algorithms.

Reference	**Power network model**	**Heuristic or theoretical method**	**Solution Methodology**	**Actuation**	**Objective**
Robbins et al. (2016)	Unbalanced SDP convex relaxation	ADMM	Open-loop	V regulation transformer tap positions	Minimize losses, total demand at substation, V magnitude deviations, Q injection to feeder bus head
Maknouninejad and Qu (2014)	Nonlinear bus injections	Gradient descent with explicit gradient term calculations	Open-loop	DG Q injection, capacitor bank switches, LTCs	Minimize quadratic voltage errors at DGs and critical nodes
Magnusson et al. (2019b)	LinDistFlow	Dual ascent with primal update	Open-loop	Q injection from PVs	Minimize quadratic Q injection/absorption cost
Schiffer et al. (2016)	Lossless bus injection	Consensus-based feedback control	Feedback-based	Inverters and P/Q sharing from DG, BESS	V magnitude setpoint
De Din et al. (2022)	Nonlinear (PYPOWER)	Primal descent-dual ascent	Feedback-based	DG Q, BESS P	Power losses
Fu et al. (2022)	Linearized DistFlow based on V sensitivities	LP using Perturb and observe power flow method for V sensitivities	Feedback-based	EVs, PVs, controllable loads	Minimize P/Q dispatch costs
Gebbran et al. (2022)	Nonlinear bus injection	Multiperiod MINLP with Consensus-based ADMM	Open-loop	PV inverter Q injection	Social welfare considering both network-level and prosumer objectives

The work (Zhao et al., 2018b) defines a new variable $U_i = V_i^2$ for all i and then uses a first-order Taylor series expansions around an operating point to linearize the power flow equation. In Capitanescu et al. (2014), the authors assume a radial topology and use the reformulation proposed in Gómez Expósito and Romero Ramos (1999) which defines a set of new variables $U_i = V_i^2$, $W_{ij} = V_iV_j\cos(\theta_i - \theta_j)$ and $T_{ij} = V_iV_j\sin(\theta_i - \theta_j)$ and rewrite (2) as a set of n quadratic and $2n$ linear constraints

$$\begin{aligned} U_iU_j =& W_{ij}^2 + T_{ij}^2 \\ P_i =& \sum_{k=1}^{n} G_{ik}U_i - G_{ik}W_{ij} - B_{ij}T_{ij} \\ Q_i =& \sum_{k=1}^{n} -(B_{ij} + b_{ij}^{\text{sh}})U_i + B_{ij}W_{ij} - G_{ij}T_{ij} \end{aligned}$$

where G_{ik}, B_{ik} and B_{ij}^{sh} are, respectively, the conductance, susceptance, and shunt susceptance of the distribution line connecting buses i and k. The work Schiffer et al. (2016) assumes a lossless network with angle difference between buses to be negligible and uses the reduced model

$$Q_i = |\hat{B}_{ii} + \sum_{k=1}^{n} B_{ik}|V_i^2 - \sum_{k=1}^{n} |B_{ik}|V_iV_k$$

where $\hat{B}_{ii}$ is the inductive shunt susceptance at node i. Some papers use simplified versions like the linearized *LinDistFlow* model, which neglects line losses and assumes radial network topology (Yeh et al., 2012; Arnold et al., 2016; Liu et al., 2017; Tang et al., 2019). Mathematically, LinDistFlow equation (Baran and Wu, 1989b,a), coupling $\mathbf{p}$ and $\mathbf{q}$ to distribution grid voltages is given by

$$\mathbf{v} = R\mathbf{p} + X\mathbf{q} + \mathbf{1}v_0$$

where v_0 denotes the voltage at the Transmission-Distribution (T-D) substation, and R, X are vectors where

$$X_{ij} = 2 \sum_{(h,k)\in\mathcal{E}_i\cap\mathcal{E}_j} x_{hk}, \quad \text{and} \quad R_{ij} = 2 \sum_{(h,k)\in\mathcal{E}_i\cap\mathcal{E}_j} r_{hk}$$

where $\mathcal{E}_i \subset \mathcal{E}$ is the set of edges in the path from Bus 0 to Bus i. A slightly different model based on fixed point methods is also used in some works (Ding et al., 2020). This realization is applicable to unbalanced as well as meshed networks and is given by

$$\mathbf{v} = \mathbf{\Phi_p p} + \mathbf{\Phi_q q} - \mathbf{Y}_{11}^{-1}\mathbf{Y}_{10}v_0$$

where $\mathbf{Y}_{11}$ and $\mathbf{Y}_{10}$ are the elements of $\mathbf{Y}$ partitioned as

$$\begin{aligned} \mathbf{Y} =& \begin{bmatrix} \mathbf{Y}_{00} & \mathbf{Y}_{01} \\ \mathbf{Y}_{10} & \mathbf{Y}_{11} \end{bmatrix} \quad \text{and} \\ \mathbf{\Phi_p} =& \mathbf{Y}_{11}^{-1}\operatorname{diag}(\hat{\mathbf{v}}^*)^{-1} \\ \mathbf{\Phi_q} =& -j\mathbf{Y}_{11}^{-1}\operatorname{diag}(\hat{\mathbf{v}}^*)^{-1} \end{aligned}$$

with $\hat{\mathbf{v}}$ being the given solution of the power flow equation around which the linearization is done.

Some works consider linear dynamical models where the incremental voltage variations are given by the product of the control inputs and the voltage sensitivity matrix (Valverde and Van Cutsem, 2013; Yorino et al., 2015; Weckx and Driesen, 2016; Robbins et al., 2013; Zhao et al., 2018a). For example, the work Valverde and Van Cutsem (2013) writes the voltage dynamics as

$$\mathbf{v}(\tau+1) = \mathbf{v}(\tau) + \frac{\partial \mathbf{v}}{\partial \mathbf{u}} \Delta \mathbf{u}(\tau)$$

where τ denotes the discrete time index and $\mathbf{u}$ denotes the control vector consisting of $\mathbf{p}, \mathbf{q}$ and tap settings of transformers with $\Delta \mathbf{u}$ as the incremental change in input. The sensitivity matrix $\frac{\partial \mathbf{v}}{\partial \mathbf{u}}$ is extracted from an offline power flow calculation. Piece-wise linear representations of the quadratic terms in the AC power flow equations are used in (Mansourlakouraj et al., 2021).

Some works do consider the exact nonlinear nonconvex AC power flow models, either using numerical methods like Newton-Raphson, Gauss-Seidel, or backward-forward sweep load flow methods (Zhang et al., 2016), or simulating the system using software tools like `DigSilent PowerFactory` (Pachanapan et al., 2012), GridLAB-D (Singhal et al., 2019), `PSCAD` (Mokhtari et al., 2013) or `OpenDSS` (Nowak et al., 2020). Finally, a few papers, such as Ding et al. (2018) use heuristic tools like particle swarm optimization (explained later in Section 3.3) or feedback-based approaches (cf. Section 3.6.2) to deal with the complexity of exact power balance equations. Selected papers also extend these models to the more realistic cases of unbalanced three-phase (Ghosh et al., 2014; Zafar et al., 2016; Shen and Baran, 2013; Haider and Annaswamy, 2022) and meshed networks (Yu et al., 2012; Schiffer et al., 2014).

3.2. Coordination Mechanisms

Depending on the decision-making or computation capabilities of various resources and the underlying communication structure, the coordination mechanisms for voltage control algorithms could be classified into the following major categories (see Figure 2 for schematics of all coordination mechanisms).

3.2.1. Local

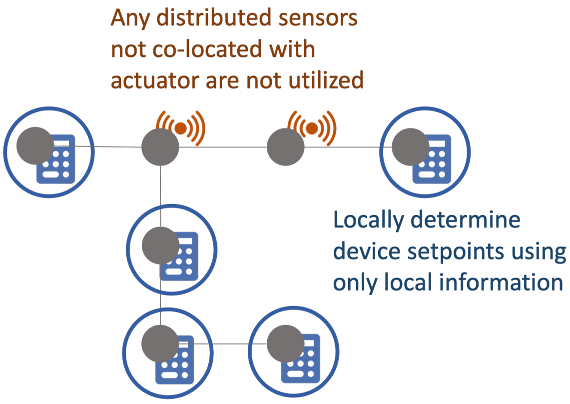
In local coordination, each agent (a reactive power resource located at a node) in the network updates its state based on its own measurement and local computation, without any external information from the network, cf. Fig. 2(a). Since there is no communication involved, these approaches are really fast, inherently inexpensive, and robust to any communication failures. However, limited use of information often results in convergence to suboptimal solutions. In fact, sometimes, these algorithms might fail to converge to a feasible solution, even if one exists, cf. Bolognani et al. (2019). In a local scheme, there might be a limited communication network and sensors deployed for the purposes of situational awareness only. The measurements may be transferred to a data historian (typically centralized), but there are no actuation signals or setpoints being generated from the sensory data.

3.2.2. Centralized

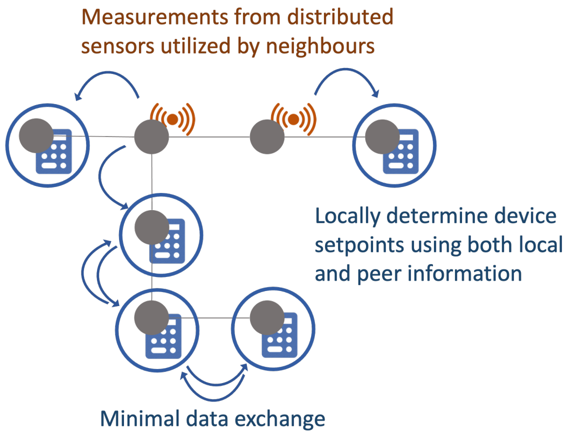
In this mechanism, there is a central coordinator or network supervisor that communicates with all the agents in the network. As shown in Fig. 2(c), the coordinator has access to all the network measurements needed, and computes the setpoints which are then communicated to all the agents. Centralized decision-making has been the standard practice in the traditional paradigm of power systems. However, with the increasing penetration of DERs, this will become intractable simply due to the amount of data that needs to be communicated and stored at the central server. Concerns about the data privacy of end users are also driving operators away from this scheme. In addition, the presence of a single point of failure in both communication and computation makes these schemes less desirable from a robustness point of view. Typically, the communication networks for the centralized schemes are designed to be point-to-point, with individual communication links between the leaf nodes or the agents and the central controller, however, meshed architectures might also be deployed. With point-to-point networks, the communication medium could vary from using power line carrier (PLC) based communication, or using Ethernet communications. In some deployments, Wi-Fi or 5G communications are also used, and these networks are typically meshed or will have multiple "hops" between the central controller and actuators. Point-to-point networks are more frequently seen in rural areas, while meshed systems are common in microgrids or dense areas. LTE and 5G communication mediums are commonly found in both areas, unlike Wi-Fi.

3.2.3. Distributed

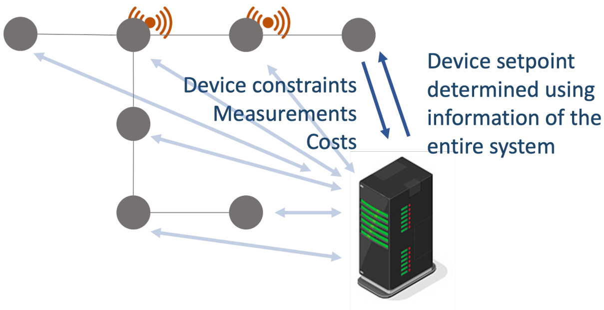
In distributed coordination, each agent in the network updates its state based on its own state, communication with its neighbors, and local computation, cf. Fig. 2(b). Distributed algorithms preserve the privacy of the participating agents and provide *plug-and-play* capabilities. The lack of a single point of failure naturally makes distributed mechanisms more robust than centralized ones. Distributed algorithms might converge slower than their centralized counterparts due to repeated data exchange. However, when designed efficiently, the memory requirement for distributed algorithms scales well with the size of the network or the number of devices. Since distributed coordination is not yet widely deployed in the field, communication architectures are still evolving. In general, these architectures are adopted more commonly in systems with multiple control devices or intelligent electronic devices (IEDs), so the communication architecture is usually meshed, and wireless. The meshed nature of the system increases the redundancy in the communication links and



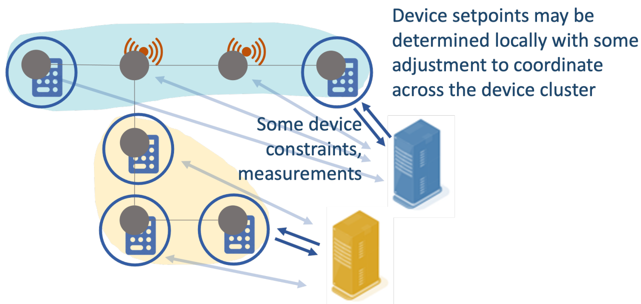
(a) **Local:** Local decision making only. No communication between nodes or any other coordinating agent. Intelligence at the resource nodes is present. Cannot leverage sensor information not collocated with a resource.



(b) **Distributed:** Bidirectional communication between neighboring agents. High degree of intelligence at the resource nodes, used to make decisions and communicate with neighbors. No central agent is required to coordinate across the network. Both synchronous and asynchronous communication models can be used.



(c) **Centralized:** Bidirectional communication between central decision making agent and each resource in the grid. Typically synchronous communication in lock-step. Limited intelligence at the resource nodes.



(d) **Decentralized:** Nodes are organized into clusters, within which their actions are coordinated by a central agent. Each node has some decision making capabilities, with the central agent providing some cluster-wide computation and coordination capabilities.

Figure 2: Different types of coordination mechanisms used for voltage control.

provides fail-safes in case of node or link failures. Majority of the works in this category rely on either the primal-dual or dual update techniques described in Section 3.3 under theoretical methods (Robbins and Domínguez-García, 2016; Qu and Li, 2020; De Din et al., 2022; Romvary et al., 2022). Interested readers are also referred to Yang et al. (2019); Nedić (2015) for comprehensive survey of works on distributed optimization.

3.2.4. Decentralized

Decentralized coordination sits in between the centralized and distributed. In this mechanism, there exist decision-making capabilities at both the local device level and a centralized coordinator. While a few works consider each entity (DER, or node) as an agent (Fusco et al., 2021; Baker et al., 2017), most of the surveyed decentralized papers use a clustering approach where the entities are aggregated into clusters based on the geographic location or other important characteristics. Each of these clusters or zones is represented by an agent in the global centralized optimization (over all clusters in the network) while performing the local optimization and coordination over devices within their respective cluster. The central coordinator is tasked with facilitating limited data exchange across the network; notably, it has access to some network data but it is not enough to compute the setpoints for all the agents. Each agent computes its own setpoints based on its own state and any network-wide information received from the central coordinator, cf. Fig. 2(d). In this way, the decision making resides at the local level but leverages some network-wide information. To some extent, decentralized algorithms preserve the privacy of the participating agents and scale well with the number of agents. However, they still suffer from a single point of failure from the use of the central coordinator. The communication architectures for decentralized coordination are still evolving, similar to the distributed approaches.

In each coordination mechanism (centralized, distributed, and decentralized), a local decision making process can also be implemented as a backup in the event of communication failure. Note that although the focus of this paper is particularly on voltage control, the coordination mechanisms defined are equally applicable to frequency control.

Remark 1 (Hierarchical Coordination). *Power systems are multi-layered cyber-physical systems, meaning that to ensure proper functioning, there are coordinators in place at various voltage levels. This naturally gives rise to hierarchical coordination, where the objectives at different levels are in principle, different from each other. As a result, the optimization algorithms used are also usually different and in fact, follow different coordination approaches. Thus, there are several works that combine two or more of the above-mentioned coordination methods, such as Pachanapan et al. (2012); Bahramipanah et al. (2016); Bidgoli and Van Cutsem (2018); Nowak et al. (2020); Mokhtari et al. (2013); Xu et al. (2022); Yu et al. (2012); Liu et al. (2019); Robbins et al. (2013); Valverde et al. (2019) included in Table 1 along with details about the constituting coordination mechanisms.*

3.3. Heuristic and Theoretical Methods

This section describes the methods used to solve the voltage optimization problem (1). These can broadly be classified into (i) heuristic-based methods and (ii) theoretical methods with convergence guarantees. We describe a few typical methods in further detail below; all other methods are noted in Tables 1 and 2.

Heuristics-based methods include classical droop-based techniques where the reactive power is injected into the network buses according to predefined Q-V curves. For example, Ghosh et al. (2014) combines the droop-based reactive power control with active power curtailment techniques. Utilizing active power curtailment expands the range of reactive power capability and leads to better voltage regulation. Some works (Baker et al., 2018; Zhou et al., 2016b) also focus on the tuning of associated droop curves characteristics to improve the voltage profile. Zhou et al. (2016b) uses day ahead forecast to tune the parameters of the Q-V curve, whereas Baker et al. (2018) updates the parameters of the Q-V as well as P-V characteristic curves based on load and solar generation forecast every 5-15 minutes. Another heuristic optimization method is the particle swarm optimization (PSO) (Clerc, 2006) used in Zafar et al. (2016); Ding et al. (2018); Zhao et al. (2018a). PSO is a computational technique inspired from the social behavior of bird flocks and can directly handle the non-linearities in the power flow equation making it a suitable candidate for solving the regulation problem (1). In PSO, the network is partitioned and each particle moves in the search space with a velocity v which depends on its own previous best solution p_{best} and its group's previous best solution g_{best}

$$\begin{aligned} v_i(\tau+1) =& w \times v_i(\tau) + c_1 \text{rand}[0,1](p_{\text{best}} - x_i(\tau)) \\ &+ c_2 \text{rand}[0,1](g_{\text{best}} - x_i(\tau)) \\ x_i(\tau+1) =& x_i(\tau) + v_i(\tau+1) \end{aligned}$$

where w is the weight matrix and rand$[0,1]$ denotes a random number between 0 and 1. Some works also use the proportional-integral (PI) method, where in addition to the proportional control law of droop-based approaches, an additional term depending on the historical performance is also added. For example, Nowak et al. (2020) uses the update rule

$$Q_i(\tau+1) = k_{p,i}\Big(V_{i,\text{set}} - V_i(\tau) + k_{I,i} \sum_{l=0}^{\tau-1} (V_{i,\text{set}} - V_i(l))\Big)$$

where $k_{p,i}$ and $k_{I,i}$ are respectively the proportional and integral gains of the controller and $V_{i,\text{set}}$ is the desired voltage for node i.

On the theoretical side, majority of the works use either the projection operator to deal with the constraints or define additional dual variables to handle them smoothly. The latter gives rise to the Lagrangian function for which various primal-dual or dual ascent methods are proposed, and are amenable to distributed implementation. Zhu and Liu (2016) uses a *projected gradient-descent (GD)* method, which combines the update at the last iterate and the projected gradient update as

$$\mathbf{q}(\tau+1) = [1-\alpha(\tau)]\mathbf{q}(\tau) + \alpha(\tau)\mathbb{P}[\mathbf{q}(\tau) - d\nabla f(\mathbf{q}(\tau))]$$

where $\alpha(\tau) \in (0,1]$, d is the stepsize and $\mathbb{P}$ denotes the projection on the feasible set. The work Liu et al. (2017) also uses a projected GD based update rule

$$\mathbf{q}(\tau+1) = \mathbb{P}[\mathbf{q}(\tau) - dD\nabla f(\mathbf{q}(\tau))] \tag{5}$$

where D is a diagonal matrix which is designed to accelerate the speed of convergence. Some works consider non-differentiable objective functions and replace the gradient term with *subgradient* in the standard projected GD (with $D = I$) in equation (5) above (Zhou et al., 2021, 2016a). A wide variety of methods are based on constructing the Lagrangian function of the constrained problem (1). In general form, *Lagrangian* of (1) is given by

$$\mathcal{L} = f(x) + \mu^\top h(x) + \lambda^\top g(x) \tag{6}$$

where μ, λ are Lagrange multipliers. The work of Shen and Baran (2013) first uses a rule-based search to find a feasible initial point using system operating conditions at a certain time. Then a reduced Lagrangian obtained after removing the term corresponding to the inequality constraints from (6) is used to compute the gradient and the corresponding update for the reactive power setpoint iteratively. Some works use *primal-dual* method which involves gradient descent in the primal and gradient ascent in the dual direction. Compared to the standard primal-dual method, Liu et al. (2019) uses a partial primal-dual method, where some of the primal updates replace the gradient descent term with a direct minimization term. *Dual ascent* which applies gradient ascent of the Lagrangian to update the dual variables and computes the $\mathbf{q}$ updates using the updated dual variables is also commonly used to solve (1) (Li et al., 2014; Bolognani et al., 2013; Magnússon et al., 2020; Bolognani et al., 2015; Ortmann et al., 2020). In Zhou et al. (2018), dual functions of the Lagrangain (6) and a reduced Lagrangian, where a subset of devices (slow devices with discrete decision variables) are treated as constant, are constructed. Then gradient ascent of the dual function is used to update the dual variables and the primal variables are updated via solving a reduced minimization problem. In updating the primal variables, two timescales are considered for devices of different updating frequencies. Dual ascent method is amenable to distributed implementation and if the problem is formulated appropriately (e.g., with separable objective function where the overall objective function is given by the summation of the objective function of all the agents), could be implemented in parallel over the distribution network. This gives rise to the *dual decomposition* method used in Wu et al. (2017); Bolognani et al. (2019); Magnusson et al. (2019a); Weckx et al. (2014). A majority of works rely on the *alternating direction method of multipliers (ADMM)* (Li et al., 2020). ADMM is based on modifying dual decomposition to attain faster convergence while utilizing its decomposability. In ADMM, the decision variable is partitioned into two sets, and the optimization problem is formulated to include just the equality constraints. The inequality constraints are dealt by incorporating them as penalty terms in the objective or via projections. The algorithm then consists of first constructing the *augmented Lagrangian* by adding an extra penalty term to (6) for violation of the equality constraints, and then applying dual ascent coupled with sequential minimization of the primal variables. As with dual decomposition method, if the optimization problem and constraints are formulated appropriately, ADMM could be implemented in a distributed fashion (Zhang et al., 2018; Liu et al., 2018; Gebbran et al., 2022). For example, Gebbran et al. (2022) partitions the original problem into subproblems and creates copies of the coupling variables. Consensus-based tools are also commonly used in the design of other distributed algorithms. The works Schiffer et al. (2016, 2014) go beyond the simplistic droop control and propose consensus-based reactive power sharing strategies. In Zhang et al. (2015), the proposed method comprises of a local and a consensus stage. In the former stage, each node solves its own problem, whereas in the latter, neighboring nodes exchange Lagrange multipliers obtained from the solutions to their corresponding local problems.

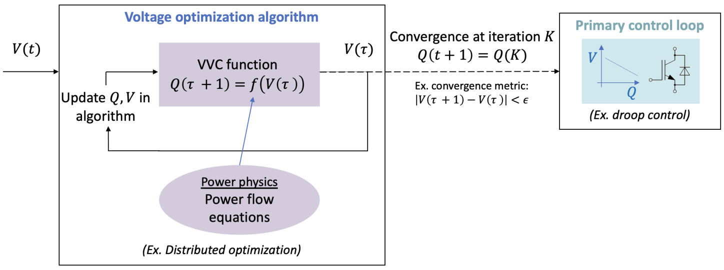
3.4. Actuation

A variety of control devices are used to perform voltage regulation by implementing the solved setpoints and command signals. These include conventional grid devices like load tap changers (LTC), which allow substation transformers to vary their turn ratio (Capitanescu et al., 2014; Valverde and Van Cutsem, 2013; Tasnim et al., 2023), as well as in-line or step voltage regulators (SVR) located in the middle of the line (Zafar et al., 2016; Kawano et al., 2016), and shunt capacitor banks. Some works also use other legacy devices like switches, circuit breakers (Capitanescu et al., 2014) and solid-state transformers (Shen and Baran, 2013; Shah and Crow, 2016). However, these electro-mechanical devices were not designed to handle the new level of variability that comes with high penetration of intermittent DERs. As such, majority of the works studied here utilize DERs (Xu et al., 2020a; Liu et al., 2017) to provide voltage regulation services. These include distributed generation (DG) (Wei et al., 2016; Li et al., 2020) such as solar photovoltaic (PV) (Ghosh et al., 2014; Zhao et al., 2018a) and battery energy storage systems (BESS) (Pachanapan et al., 2012; Bahramipanah et al., 2016) - controlling their P and Q injections through smart inverters and variable power factor control. Note that the design and control of inverters is a significant area of research, with pertinent research directions in the design of grid-forming and grid-following inverter design (Lin et al., 2020). Details of this research area is outside the scope of this paper. In addition to the aforementioned devices, controllable loads like electric vehicles (EV) (Mansourlakouraj et al., 2021; Weckx et al., 2014) and thermostatic loads (heating, ventilation, and cooling) (Zhou et al., 2018) may also be used in demand response (DR) (Zhang et al., 2015; Magnússon et al., 2020; Haider and Annaswamy, 2022) programs for voltage support.

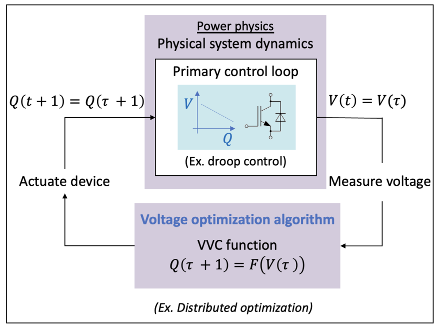
3.5. Objectives

The works studied here achieve voltage regulation by considering various objectives in (1). These include first and foremost, voltage regulation, and classified into: (i) keeping bus voltage magnitudes within the standard limits[1] by penalizing violations, (ii) maintaining voltages close to the nominal setpoint or reference value (generally 1 p.u.), and (iii) achieving a flat nodal voltage profile over the distribution network by minimizing fluctuations. Several works also aim to minimize (iv) line losses, (v) power imports from the bulk grid at the transmission-distribution (T-D) substation (or distribution transformer), and (vi) active power curtailment from flexible loads and/or renewables like solar PV. Another common objective is to (vii) minimize reactive power dispatch, i.e., absorption or injection by smart inverters, capacitor banks, etc. for voltage support and Q compensation. For conventional grid devices described in Section 3.4 above, the objective is (viii) to minimize the tap operations and switching of capacitors and LTCs. Finally, (ix) conservation voltage regulation (CVR) can also be implemented, in which case the grid operator attempts to minimize voltage magnitudes across the network to achieve energy and demand reductions as well as costs for end-use customers.

[1] Voltage standards detailing the allowable deviation from nominal voltage under normal grid conditions vary globally. North America follows ANSI C84.1 which allows $\pm 5\%$ deviation, while Europe follows IEC and European EN 50160 which allows $\pm 10\%$ deviation. Thus the minimum voltage in Europe is 0.9 per unit (p.u.), and in North America is 0.95 p.u.



(a) **Open-loop:** the solution (i.e., Q setpoint) is implemented only after the algorithm converges. The power flow equations are used in designing the algorithm update rules (or the function f).



(b) **Feedback-based:** the Q setpoint is implemented at every iteration of the computation scheme. In doing so, feedback from the physical system by way of voltage measurement is used in updating Q at the next iteration/time instant.

Figure 3: Open-loop and feedback-based solution methodologies for voltage control.

3.6. Solution Methodology

In this section, we present an important classification of the solution methodology of voltage control algorithms based on how the network measurements are utilized in the algorithmic updates. Note that this classification is most relevant for iterative distributed algorithms (shown in Table 2) that repeatedly use values obtained from either network model calculations or actual system measurements at each iteration.

3.6.1. Open-loop

In the *open-loop* or *offline* methodology shown in Figure 3(a), the algorithmic updates are computed using a model of the distribution system and the setpoints are implemented only after the algorithm converges or a certain stopping criterion (number of iterations/desired accuracy) is met. This approach might suffer from imperfect knowledge of the network parameters and in general, is only applicable for static voltage control, where the load remains constant throughout the algorithm evolution. Since no measurements are used in the updates, the speed of updates in the open-loop approach is only constrained by the communication bandwidth, although the convergence speed to attain a desired level of accuracy depends on the specific algorithm employed. Details of the workflow of a specific example are shown in Figure 4.

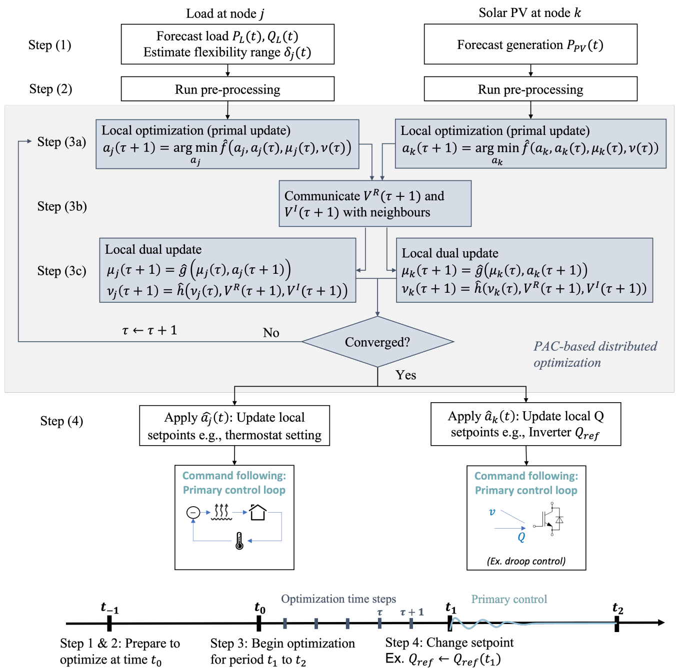


Figure 4: Example workflow for an **open-loop** distributed optimization method, illustrating the Proximal Atomic Coordination algorithm from Romvary et al. (2022); Haider and Annaswamy (2022).

3.6.2. Feedback-based/Closed-loop

The merits of feedback control have given rise to another solution methodology for voltage control algorithms, where the setpoints are implemented in parallel with the algorithm execution by using measurements. The use of measurement data gives rise to the *feedback-based* or *closed-loop* approach. In this data-driven approach, in order to avoid dealing with the nonlinear power balance equation (2), voltage measurements from the system are used to compute the updated setpoints at every iteration, cf. Figure 3(b). Using measurements helps to overcome many of the computational burdens inherent in the AC power flow equations (2), and communication requirements of open-loop approaches. Importantly, this method allows a quasi-static treatment as the new power flow information due to changing load and generation is captured via measurements. The update frequency in this feedback approach is usually constrained by the rate at which new measurements are available. It should be noted that since the local coordination relies on agents' local measurements for updating their states, it is mostly feedback-based. The most important feature of this method is the following: Measurements are used as a *surrogate or proxy for solving the power flow equations*, essentially converting the optimization problem (1) into a feedback control problem,

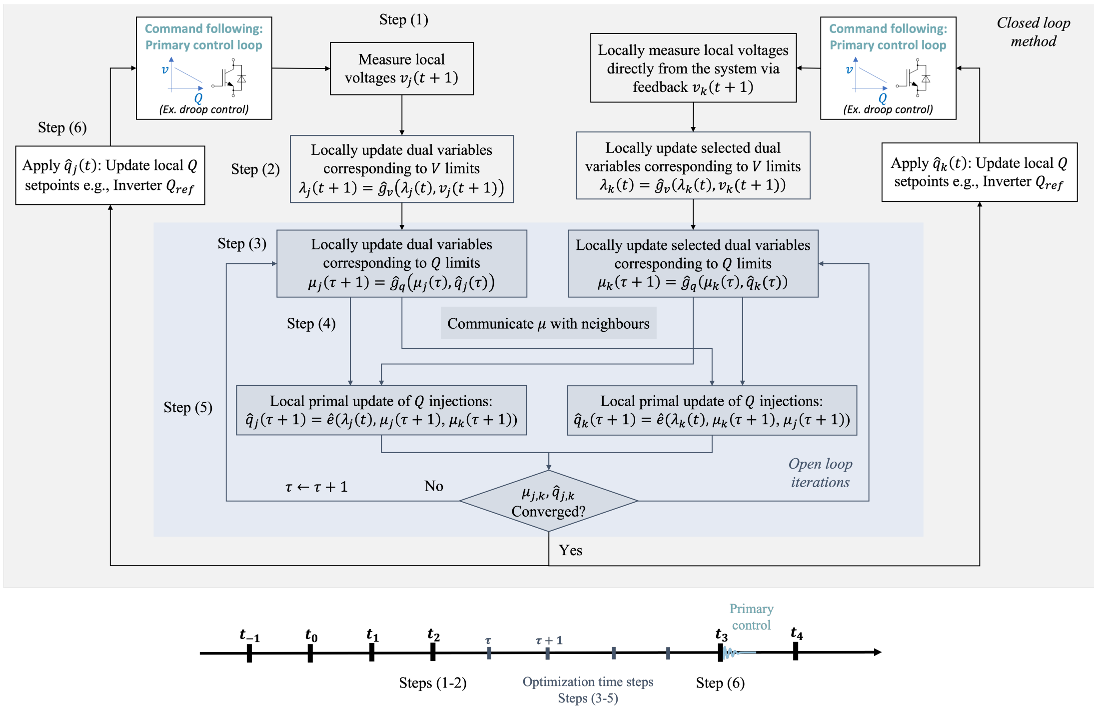


Figure 5: Example workflow for a distributed optimization method with **feedback-based** methodology, based off of Bolognani et al. (2019); Ortmann et al. (2020). Note that while the open-loop approach in Figure 4 uses both P and Q injections as decision variables or actuation inputs, the feedback-based approach here only uses Q injections.

where the dynamics show up in the controller, but due to the fast timescale of power electronics, the plant is represented as an algebraic model leading to a flipped control problem. One should be cognizant that viewing the plant-model as algebraic may not be appropriate if the measurements vary at a fast time-scale. The latter is quite possible with advances in internet-of-things (IoT) networks causing new information at fast time-scales to be increasingly available. Details of the workflow of a specific example are shown in Figure 5.

3.6.3. Examples

Figures 4 and 5 illustrate two algorithmic implementations of the methods described in Section 3.6.1 and Section 3.6.2. Figure 4 shows an open-loop approach, where the updated setpoints are used for actuation only after the algorithm has converged in the open-loop scheme. On the other hand, in the feedback-based example shown in Figure 5, the iterates are used for actuation once a subset of the variables has converged in the inner open loop, and the resulting measurements are used to update the remaining variables in the feedback-based outer loop at every timestep. In Figure 5, a subset of the dual variables (λ) is updated only once per timestep t (in the feedback loop using the V measurements). But the primal variables ($\hat{q}$) and remaining duals (μ) are updated at each iteration τ in an open-loop fashion, until convergence. Furthermore, the feedback-based approach only utilizes real-time operational data and measurements, while the open-loop method also relies on forecasts. We would like to remind the reader that these are just examples and that the exact structure of the algorithms may vary. For instance, some papers employing the feedback-based methodology such as Magnússon et al. (2020), are purely feedback-based i.e. they don't have any inner open-loop iterations, and only use measurement-based updates at each timestep. Other feedback-based approaches may also replace local controllers, i.e., no longer use droop curves. The timescales for feedback-based approaches differ according to the underlying power system and technologies used. For legacy distribution systems without DERs, voltage control typically happens in the scale of minutes. For future distribution systems with higher penetration of DERs, it is expected that voltage control needs to happen more frequently, for example, every 10 seconds.

The figures above highlight the key differences between the open-loop and feedback-based schemes. In the feedback-based scheme shown here, the use of voltage measurements allows us to solve the problem without having to explicitly enforce the power flow equality constraint h. In addition, the dual variables are also calculated using the inequality constraints $\hat{g}_q$ and $\hat{g}_v$ corresponding to the Q and V limits, respectively. This allows to readily compute Q injections via an algebraic function $\hat{e}$, and actuate (update Q setpoints) and measure voltages more frequently at each t. This is indicated visually by the condensed timeline in Figure 5. However, we have magnified one of the timesteps $[t_2, t_3]$ to illustrate the faster open-loop iterations τ within each interval.

4. Implementation Barriers and Future Research Directions

In this section, we identify a few key implementation barriers and outline some future research directions.

4.1. Model Accuracy: Load Modeling

The proposed voltage control mechanisms typically assume PQ load models – constant real power and constant reactive power models – to represent the loads connected in the distribution grid. However, the PQ model does not capture the sensitivity of loads to voltages, which is of particular importance to voltage control in the distribution grid. The ZIP load model (constant impedance, constant current, and constant power) in eqs.(7a)-(7b) and exponential load model (direct function of voltage) in eqs.(8a)-(8b) have been used to characterize this sensitivity.

$$P(V) = P_0 \left(Z_p \left(\frac{V}{V_0} \right)^2 + I_p \left(\frac{V}{V_0} \right) + P_p \right) \tag{7a}$$

$$Q(V) = Q_0 \left(Z_q \left(\frac{V}{V_0} \right)^2 + I_q \left(\frac{V}{V_0} \right) + P_q \right) \tag{7b}$$

where $Z_p, I_p, P_p, Z_q, I_q, P_q$ denote the proportional coefficients in the percentage of the constant impedance, constant current and constant power in static active and reactive load.

$$P(V) = P_0 \left(\frac{V}{V_0} \right)^{n_p} \tag{8a}$$

$$Q(V) = Q_0 \left(\frac{V}{V_0} \right)^{n_q} \tag{8b}$$

where the exponents n_p, n_q are the model parameters. The ZIP load is frequently used in load modeling, but it cannot be easily accommodated in many convex relaxations of optimal power flow problems due to the constant-current component's linear dependency on the voltage magnitude (Shen et al., 2019). Various approximations to the ZIP model have been proposed for SOCP, SDP, and QP relaxations of the OPF (Shen et al., 2019; Molzahn et al., 2014; Claeys et al., 2021), but there is limited integration of these models with proposed voltage control methods. Another approach is to use polynomial approximations for ZIP models using best-fit coefficients (Ozdemir and Baran, 2020). Some works propose voltage control methodologies that account for voltage-dependent loads using a linear combination of constant Z, I, and/or P models, (Claeys et al., 2021; Shah and Crow, 2016) whereas other papers are proposed and tested using constant power (PQ bus) load models (Jelani et al., 2013; Schiffer et al., 2014; Ding et al., 2017). PQ models are also a reasonable approximation for certain microgenerators (Bolognani et al., 2013). In some cases, microgenerators and loads use exponential models instead (Bolognani and Zampieri, 2013; Wang et al., 2014). DER buses with PV and smart inverters are generally modeled as PV buses such as in (Cagnano and De Tuglie, 2015). The above methods can be applied to both delta- and wye-connected lods with suitable modifications. Further, conventional load modeling methods derive model parameters using spot measurements that may not be able to capture time-varying load behaviors (Arif et al., 2018). Characterizing time-varying loads and accurately modeling them is essential to developing and testing voltage control methodologies, especially if the deployment is open-loop. Offline testing with representative load data can accelerate online testing and deployment, reducing barriers to adopting new methodologies in the field. Lastly, the slack bus at the point of common coupling (PCC) or substation is usually modeled as a constant voltage generator. The control objective also influences the choice of load model to some extent, e.g., polynomial models may be better suited if we consider CVR applications in addition to voltage support (Ozdemir and Baran, 2020). Another related topic of crucial importance for efficient voltage control is accurate load forecasting, especially with the surge in machine learning tools. We do not go into the specifics of forecasting methods and open problems in this direction; details can be found in Hong and Fan (2016); Kuster et al. (2017); Yildiz et al. (2017); Haben et al. (2021).

4.2. Validation and Comparison: Data Availability

The lack of standardized test cases makes validation and comparison of proposed methods difficult. The National Renewable Energy Laboratory Workshop on Autonomous Energy Grids (2017) concluded that "A major limitation in developing new technologies for autonomous energy systems is that there are no large-scale test cases ($\cdots$). These test cases serve a critical role in the development, validation, and dissemination of new algorithms". Real network data is not available due to data privacy concerns and system security[2]. Historically, the IEEE has developed and released test networks to overcome this issue; typical test systems used in the literature consist of the radial IEEE distribution test feeders (Kersting, 2001, 2006), such as the IEEE-8, 13, 4, 37, and 123 bus systems. A few works have also tested their algorithms on real distribution feeders such as those of Southern California Edison (SCE) (Zhou et al., 2016b)) as well as developing quasi-static time-series distribution systems using real utility data from smart meters (Bu et al., 2019). These test networks are suitable for preliminary testing, but do not provide sufficient opportunity for comparing methods or moving towards technology deployment, as discussed below:

(i) **Lack of DERs:** The existing IEEE feeders do not contain DER data. Instead, each researcher creates

[2]This is an established and known concern. To this end, recent work has focused on developing models of real systems, including a recent open-source model of the contiguous US grid (Xu et al., 2020b).

their own test case with varying DER penetration, characteristics, and models.

(ii) **Load conditions:** The IEEE test cases have load data for only a single timestep and do not provide any representative load profiles. To enable accurate simulations including adjustment of volt-var control equipment and dispatch of DERs for voltage control, load and generation profiles are central components of the data and model.

(iii) **Network size:** The IEEE test feeders are often too small to show demonstrable performance for realistic distribution networks, and in particular for the distributed and decentralized test cases. The lack of representative large-scale models of utility distribution systems with hundreds to thousands of nodes and high DER penetration prevents testing the computational tractability of distributed volt-var control (VVC) methods. The IEEE 8500 node feeder was created to test algorithm scalability but still suffers from (i) and (ii).

(iv) **Computing and hardware infrastructure:** The power grid is a cyber-physical system and proposed decision making schemes rely heavily on a combination of sensing, widespread communication and computation, and actuation. The interactions of physical power hardware and digital computations require testbeds with these capabilities integrated into a single platform – often termed "hardware-in-the-loop" (HIL). However, HIL testbeds are not widespread, and interfacing, implementation, and testing require significant time and expertise (Venkataramanan et al., 2017). The hardware being tied in results in higher fidelity experiments, and both control hardware and power hardware devices can be included. A survey of cyber-physical testbeds is provided in (Cintuglu et al., 2017).

In response to deficiencies (i)-(iii), an extension to the 8500 test feeder is underway. The new 9500 node feeder is a representative power system model developed as a part of the GridAPPS-D™ project and will include behind-the-meter customer rooftop PV and utility-scale DERs[3]. To be most useful for testing volt-var control methodologies – key operational scenarios listed by the Working Group – the feeder data must also include different time series data for load profiles and generation, including models of residential, commercial, and industrial loads in appropriate locations of the feeder, and load profiles to characterize different phenomenon such as the duck curve[4], heat wave[5], and high intermittency in generation[6]. In response to (iv), a National Science Foundation user facility consisting of about 2500 real DERs to facilitate testing of distributed communication and controls algorithms at scale is also being developed at the University of California, San Diego[7].

4.3. Data-Driven and Model-Free Methods

The focus of this paper is largely model-based methods which employ a power systems based model. However, the growth of sensing, communication, and widespread DER adoption is transforming the distribution grid, and making available more and more data. Concurrently, the recent developments in the domain of model-free methods for safety-critical applications such as robotics and autonomous driving are renewing interest in model-free data-driven methods for power system applications, such as voltage control. The model-free approach is largely amendable to local and distributed approaches (as per the classification of coordination mechanisms in this paper). These algorithms build a model of the system by interacting either offline with a simulation model or online with the real power system. Note that while research is actively being done in this area, industry adoption of model-free methods is slow. And there is a need to come up with performance guarantees to overcome the hesitancy prevalent in utilities and system operators in adopting black box methods with little interpretability. Here we present a brief discussion of this growing body of research; a more detailed review can be found in Chen et al. (2022); Cao et al. (2020) for the use of reinforcement learning for power systems.

Early research has investigated the use of reinforcement learning (RL) applied to reactive power control (Vlachogiannis and Hatziargyriou, 2004) and power system stability (Ernst et al., 2004), largely with a focus on transmission grid operations. More recently the research direction has focused on the distribution grid where interest in RL comes from: (i) the need for plug and play capabilities due to the growing number of IBRs distributed throughout the grid; (ii) increase in data availability and sophistication in safe RL; and (iii) uncertainty or even complete lack of knowledge of the underlying distribution grid topology. Some approaches model the decision maker

[3] `https://cmte.ieee.org/pes-testfeeders/temporary/`

[4] The "duck curve" is a phenomenon experienced in solar rich regions, where high solar generation during the day results in very low net load (the belly of the duck) followed quickly by the very high net load when the sun sets and residential loads increase in the evening (head of the duck). This new operating condition, where dispatchable bulk resources must quickly meet the large and rapid change in electricity demand, introduces challenges to grid operators, including voltage management.

[5] High air conditioning loads can result in voltage drops across the network

[6] This introduces challenges with voltages sharply increasing or decreasing in response to cloud cover affecting the output of solar PV units

[7] `https://sites.google.com/ucsd.edu/derconnect`

as an agent overseeing a group of IBRs, such as the decentralized Multi-Agent Reinforcement Learning (MARL) approach (Wang et al., 2021), and distributed MARL using a consensus approach (Gao et al., 2021). Other approaches model the decision maker as an agent overseeing a single IBR, with a controller that is locally trained. In this approach, significant efforts have been made towards guaranteeing system stability by satisfying certain Lipschitz constraints (Cui et al., 2022a,b), or learning the network topology with provably finite-time convergence to safe voltage limits (Yeh et al., 2022). Finally, other works consider deep RL approaches for local control of shunt capacitors (Yang et al., 2020), control of electric vehicles to mitigate voltage violations (Sun and Qiu, 2021), and deep RL for load shedding in response to emergency voltage control situations (Huang et al., 2022).

4.4. Communication Protocols

Understanding communication infrastructure is essential to perform voltage control. An important point of interest in the communication infrastructure is the communication delay or latency, and the dependence of voltage control methods on receiving timely information. Table 3 presents a summary of communication technologies and their corresponding latencies. Presumably, a distributed voltage control approach that requires iterative information sharing between neighboring agents may not be realizable with slow communication; the communication latency may be prohibitive in time to reach an optimal decision. However, the emergence of technologies for critical IoT systems – including smart power systems – can enable faster data sharing with high reliability guarantees.

Table 3: Summary of communication latency and technology readiness levels of different communications used in (or projected for) power grid applications.

	Maximum latency	**Technology readiness**
Slow communication (Kansal and Bose, 2012)	100 ms	Past: transmission grid transient stability
Critical IoT Connectivity (Ericsson Technology Review Articles, 2020)	50 ms (99.9% reliability)	Current: Piloting in many industries
Ultra-reliable low-latency communication (URLLC) (Ericsson Technology Review Articles, 2020)	1 ms (99.9% reliability)	Future: 5G New Radio standard release target, one-way latency

In terms of the communication protocol, the most common protocols to communicate with field devices are DNP3 and IEC 61850 Manufacturing Messaging Service (MMS) protocols. With IBRs, newer protocols such as IEEE 2030.5 might be used. The communication protocols do not affect the nature of the decision making process, nor the communication architecture, as the protocols can be adapted to suit the requirements. In certain cases, the communication protocol might not support routing or might operate only in a specific layer of the TCP/IP network stack, at which point these protocols are either augmented or replaced with something more suitable.

4.5. Cybersecurity

Cybersecurity is an important consideration for voltage control, especially with increasing attack surface with a higher number of digital devices used in voltage control. For a voltage control application, the physical plant is the overall grid, the sensors are the measurement devices or IEDs that measure the voltage, the control and computation block contains the algorithm that performs the voltage control and the actuators could be legacy devices such as tap changers/capacitors or be newer IBRs. The feedback model can be used to represent all the decision making paradigms, from local to distributed, with the communication links considered according to the architecture.

For different coordination mechanisms, cybersecurity concerns can either be paramount or minimized. Local and distributed algorithms provide the most resistance to cybersecurity concerns, considering that the communication between external components is minimized, and communication that does happen usually happens internally. Even in these cases, the components themselves could be vulnerable to attacks due to third-party components from a suspect supply chain, or due to hardware-based attacks such as side-channel or fuzzing attacks from unmanned substations. In distributed algorithms, the attacker needs to compromise many devices at the same time to have a significant impact on the system, as the algorithm naturally provides redundancies against single-point failures. On the other hand, for centralized or decentralized systems, which rely heavily on communication between components for decision making, the attack surface is significantly increased. In addition to the individual components themselves being vulnerable to compromise similar to the local or distributed frameworks, the centralized and decentralized mechanisms are also vulnerable to attacks such as insider attacks, man-in-the-middle attacks, and denial-of-service attacks among others. Allegorically, the presence of a hierarchy among the components provides an advantage to the attacker, because compromising the centralized controller provides more access than compromising a leaf node. In other words, single points of failure exist in these architectures, and these resources have to be protected additionally to ensure minimum disruptions to the operation.

In the field, the measurement and control components are usually isolated within a closed network which restricts access to non-authorized devices. Defense techniques such as defense-in-depth often referred to as the Purdue model also provide segmentation to the network and ensure that devices with higher priority (also referred to as crown jewels) are better protected or isolated from more vulnerable devices. In addition, traditional IT-based security tools are also deployed in securing these devices, such as intrusion detection systems, firewalls, host and network monitoring, access control, encryption, and more. However, not all of these mitigation strategies are deployed at all installations, and these mitigation mechanisms could also be misconfigured leading to additional problems. Finally, it should also be noted that for all the coordination schemes, there are usually fail-safes deployed which will enable the devices to work in local mode if necessary when the communication fails. This capability could also be enabled over supervisory control and data acquisition (SCADA) if necessary and provides the operators a way of controlling the system sub-optimally. The final fail-safe is for the utility to dispatch maintenance crew directly to manually operate these devices as a last resort.

4.6. Industrial Practice and Implementation Challenges

Voltage control in the distribution system has been an important problem for distribution systems from the early 1960s at least (Cook, 1961), considering that consumer appliances at this time were becoming more sensitive to voltage deviations. The use of voltage regulators and tap changers apart, shunt capacitors were being used to control the voltage for both safety and economic considerations. Shunt capacitors were initially used as a permanent fixture, following which the use of switched capacitors became more prominent (Grainger and Civanlar, 1985). The economic advantages of voltage control become apparent when considering the fact that the losses are minimized when transmitting at lower voltages, due to Ohm's law. The voltages need to be regulated between 0.95 - 1.05 p.u. for safety and standardization, but for economical reasons, in CVR, the voltages are regulated more tightly, usually between 0.98 to 0.95. This minimizes the losses in the line, while still ensuring that safety standards are maintained. There have been several pilot demonstrations of CVR, with one of the earliest being the pilot demonstration from Snohomish County Public Utility District (PUD) (Kennedy and Fletcher, 1991). The results demonstrated a 2.1% voltage reduction, as well as reduced energy consumption by the same amount. Customer bills, "after a rate adjustment to accommodate fixed operations costs, were approximately $6.28 lower per customer per year", as detailed in a National Rural Electric Cooperative Association (NRECA) report studying the benefits of CVR.

While voltage control and CVR are part of the standard operating procedure in the industry, the use of DERs or distributed control components for CVR is still in its infancy. The regulation around using IBRs for voltage control is dictated by the IEEE 1547 standard and individual state requirements such as California's Rule 21.

4.7. Resilience

Resilience is becoming a key consideration with the increasing number of high impact low frequency (HILF) events due to various causes such as natural disasters, cyber-attacks, and so forth. Voltage regulation is an important component in supporting the system's ability to be resilient: some of the electrical equipment can be damaged if the voltage is not tightly regulated. As Section 3 demonstrates, distributed control schemes provide a clear advantage over centralized schemes as there is more redundancy built into the system in terms of both physical control devices and communication/computation infrastructure. Local control schemes can be coordinated to operate in case the centralized control mechanisms fail.

However, there is very little work in understanding *how much more resilient* systems become when moving from different control architectures, or if other parameters are changed. A major challenge in understanding or quantifying the increase in resilience is due to the lack of standardized resilience metrics that can be used to track system performance over time, or benchmark performance across different systems. Resilience is also important in understanding how the grid copes with the decreasing inertia, how to deal with increasing intermittency from renewable energy sources such as PV, and how the integration of third-party-owned devices such as customer-owned assets will affect voltage regulation. These challenges are outlined further in the National Academies Report on "Enhancing the Resilience of the Nation's Electricity System" (National Academies of Sciences, Engineering, and Medicine, 2017), and in the report "The Future of Electric Power in the United States" (National Academies of Sciences, Engineering, and Medicine, 2021).

5. Conclusion

With the rapid integration of DERs into distribution systems, voltage regulation schemes are evolving to adapt to the changing scenarios. This paper provides a comprehensive review of the state-of-the-art voltage control algorithms by classifying them across power network models, coordination mechanisms, heuristic and theoretical methods, actuation devices, objectives, and solution methodologies. The interactions between the physical power system, the voltage regulation scheme, and the actuation of control devices are detailed. Implementation barriers to the state-of-the-art algorithms are examined across model accuracy, data availability for validation and comparison, performance guarantees for model-free methods, communication and cybersecurity considerations for advanced, communication-dependent schemes, and resilience. We hope that this detailed examination spurs future research in relevant directions and eventually widespread adoption by the system operators and industrial partners.

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